# Number Theory in Fukuoka

Organizers: Masato Wakayama (NTT/Kyushu University) Yasuro Gon (Kyushu University)

Date : November 9, 2023 – November 10, 2023 Venue : IMI Auditorium W1-D-413, West Zone 1 Kyushu University, Fukuoka, JAPAN.

## **Titles and Abstracts**

### Nov.9 (Thu)

9:45-10:45 Takuki Tomita (Keio University)

Title: Absolute zeta functions arising from ceiling and floor Puiseux polynomials

Abstract: Deitmar-Koyama-Kurokawa calculated the absolute zeta function of the  $\mathbb{Z}$ -lift  $X_{\mathbb{Z}}$  of a monoid scheme X of finite type by interpolating  $\#X_{\mathbb{Z}}(\mathbb{F}_q)$  for all prime powers q using the Fourier expansion. This absolute zeta function coincides with the absolute zeta function of a certain polynomial. In this talk, we characterize this polynomial as "a ceiling polynomial." Moreover, extending this idea, we introduce a certain pair of absolute zeta functions of a separated scheme X of finite type over  $\mathbb{Q}$  by using "the ceiling and floor Puiseux polynomials" of X, which estimate " $\#X(\mathbb{F}_{p^m})$ " for sufficiently large p. In particular, if X is an elliptic curve, then our absolute zeta functions of X do not depend on its isogeny class. This talk is based on a joint work with Yoshinosuke Hirakawa, Tokyo University of Science.

#### 11:00-12:00 Seiji Kuga (Sophia University)

Title: On the Rankin-Selberg integral for Siegel cusp forms for square-free level of degree 2

Abstract: Andrianov suggested the Dirichlet series constructed from the Fourier coefficients of Siegel cusp forms for  $\text{Sp}_2(\mathbb{Z})$  and proved its functional equation by computing the Rankin-Selberg integral. In this talk, we consider the Bessel periods for  $\text{GSp}_2$  for Siegel cusp forms for square-free levels and calculate the Rankin-Selberg integrals. If time permits, we will give an asymptotic formula of central values of the spinor *L*-function for Siegel cusp forms of degree 2. This is a joint work with Masao Tsuzuki (Sophia University).

13:30-14:30 Cid Reyes Bustos (NTT Institute for Fundamental Mathematics)

#### Title: Limits of the spectral zeta function of the quantum Rabi model

Abstract: In this talk we discuss limits of the spectral zeta function of the quantum Rabi model with respect to the system parameters. In particular, when the coupling strength of the system goes to infinity the limit of the spectral zeta function is just a sum of two Hurwitz zeta functions, giving a direct proof of a result obtained by Fumio Hiroshima using resolvent methods. The computations are done using the explicit formulas of the heat kernel and partition functions of the quantum Rabi model obtained by the author and Masato Wakayama in a previous study.

## 14:45-15:45 Taichi Kousaka

### Title: The Bartholdi zeta function for infinite weighted graphs

Abstract: The Ihara expression of the zeta function of infinite weighted graphs with the Fredholm determinant was established by A.Deitmar in 2015. In this talk, we introduce the Ihara zeta function of Bartholdi type for infinite weighted graphs and give the Ihara expression with the Fredholm determinant. This is a generalization of the Ihara expression of the zeta function obtained by A.Deitmar in 2015.

## 16:00-17:00 Anton Deitmar (Universität Tübingen)

#### Title: Prime Geodesic Theorems

Abstract: An overview on various Prime Geodesic Theorems in the context of locally symmetric spaces, graphs and buildings will be given. Applications to class numbers will be presented.

## Nov.10 (Fri)

## 9:45-10:45 **Shuji Horinaga** (NTT Institute for Fundamental Mathematics) Title: Cuspidal components of Siegel modular forms for large discrete series representations

Abstract: A classical structural theory of holomorphic Siegel modular forms implies the decomposition of the space of them into the direct sum of cusp forms and Eisenstein series. Moreover, if the weights are large, Eisenstein series are a sum of Siegel Eisenstein series and Klingen Eisenstein series. In this talk, we consider the automorphic forms on  $Sp_4(\mathbb{R})$  generating large discrete series representations and their structural theory. We show that if the infinitesimal characters of the representations are sufficiently regular, we prove the similar structural theorem for automorphic forms generating large discrete series representations. This is a joint work with Hiro-aki Narita.

## 11:00-12:00 Masao Tsuzuki (Sophia University)

### Title: The archimedean Bessel function and a period relation on GSp(2)

Abstract: Based on Moriyama's formula, we propose a way to normalize the split Bessel function and the Whittaker function of large discrete series type on the real symplectic similitude group. As an application, we prove a period relation between the Bessel periods and the Whittaker period of an irreducible cuspidal automorphic representation of discrete series type.