

問. ここでは掃き出し法を主に用いた解答を紹介するが, 余因子展開を効果的に用いるとより簡単に計算できることもある. いろいろ工夫してみよ.

(1)

$$\det \begin{pmatrix} 2 & -4 \\ 3 & -5 \end{pmatrix} = 2$$

(2)

$$\det \begin{pmatrix} 5 & -3 \\ 0 & -5 \end{pmatrix} = -25$$

(3)

$$\det \begin{pmatrix} 3 & -4 \\ -5 & 1 \end{pmatrix} = -17$$

(4)

$$\det \begin{pmatrix} 0 & 2 \\ 3 & 3 \end{pmatrix} = -6$$

(5)

$$\det \begin{pmatrix} -2 & 4 \\ 5 & 2 \end{pmatrix} = -24$$

(6)

$$\det \begin{pmatrix} -1 & 3 \\ 0 & 0 \end{pmatrix} = 0$$

(7)

$$\begin{aligned} \det \begin{pmatrix} 5 & -2 & 0 \\ 2 & -4 & -4 \\ 3 & -3 & -3 \end{pmatrix} &= 2 \det \begin{pmatrix} 5 & -2 & 0 \\ 1 & -2 & -2 \\ 3 & -3 & -3 \end{pmatrix} \\ &= 2 \det \begin{pmatrix} 0 & 8 & 10 \\ 1 & -2 & -2 \\ 0 & 3 & 3 \end{pmatrix} \\ &= -2 \det \begin{pmatrix} 8 & 10 \\ 3 & 3 \end{pmatrix} \\ &= 12 \end{aligned}$$

(8)

$$\begin{aligned} \det \begin{pmatrix} 4 & 3 & 3 \\ 4 & -5 & -3 \\ -5 & -1 & -3 \end{pmatrix} &= 4 \det \begin{pmatrix} 1 & \frac{3}{4} & \frac{3}{4} \\ 4 & -5 & -3 \\ -5 & -1 & -3 \end{pmatrix} \\ &= 4 \det \begin{pmatrix} 1 & \frac{3}{4} & \frac{3}{4} \\ 0 & -8 & -6 \\ 0 & \frac{11}{4} & \frac{3}{4} \end{pmatrix} \\ &= 4 \det \begin{pmatrix} -8 & -6 \\ \frac{11}{4} & \frac{3}{4} \end{pmatrix} \\ &= 42 \end{aligned}$$

(9)

$$\begin{aligned} \det \begin{pmatrix} -4 & 5 & 5 \\ -2 & -2 & 0 \\ -4 & -3 & 3 \end{pmatrix} &= -2 \det \begin{pmatrix} -4 & 5 & 5 \\ 1 & 1 & 0 \\ -4 & -3 & 3 \end{pmatrix} \\ &= -2 \det \begin{pmatrix} 0 & 9 & 5 \\ 1 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} &= 2 \det \begin{pmatrix} 9 & 5 \\ 1 & 3 \end{pmatrix} \\ &= 44 \end{aligned}$$

(10)

$$\begin{aligned} \det \begin{pmatrix} -2 & -1 & 2 \\ -4 & 5 & 3 \\ 5 & -2 & -1 \end{pmatrix} &= -2 \det \begin{pmatrix} 1 & \frac{1}{5} & -1 \\ -4 & 5 & 3 \\ 5 & -2 & -1 \end{pmatrix} \\ &= -2 \det \begin{pmatrix} 1 & \frac{1}{7} & -1 \\ 0 & -\frac{9}{2} & 4 \\ 0 & -\frac{9}{2} & 4 \end{pmatrix} \\ &= -2 \det \begin{pmatrix} 7 & -1 \\ -\frac{9}{2} & 4 \end{pmatrix} \\ &= -47 \end{aligned}$$

(11)

$$\begin{aligned} \det \begin{pmatrix} -2 & 2 & 5 \\ -1 & 1 & 4 \\ 2 & -5 & -4 \end{pmatrix} &= -\det \begin{pmatrix} -2 & 2 & 5 \\ 1 & -1 & -4 \\ 2 & -5 & -4 \end{pmatrix} \\ &= -\det \begin{pmatrix} 0 & 0 & -3 \\ 1 & -1 & -4 \\ 0 & -3 & 4 \end{pmatrix} \\ &= \det \begin{pmatrix} 0 & -3 \\ -3 & 4 \end{pmatrix} \\ &= -9 \end{aligned}$$

(12)

$$\begin{aligned} \det \begin{pmatrix} 5 & -1 & -1 \\ 1 & 4 & 3 \\ 2 & 2 & 1 \end{pmatrix} &= \det \begin{pmatrix} 0 & -21 & -16 \\ 1 & 4 & 3 \\ 0 & -6 & -5 \end{pmatrix} \\ &= -\det \begin{pmatrix} -21 & -16 \\ -6 & -5 \end{pmatrix} \\ &= -9 \end{aligned}$$

(13)

$$\begin{aligned} \det \begin{pmatrix} -3 & -4 & -2 & -3 \\ -5 & -4 & 2 & 4 \\ 0 & 1 & 1 & 2 \\ 3 & 1 & 1 & 2 \end{pmatrix} &= -3 \det \begin{pmatrix} 1 & -\frac{4}{3} & \frac{2}{3} & 1 \\ -5 & -4 & 2 & 4 \\ 0 & 1 & 1 & 2 \\ 3 & 1 & 1 & 2 \end{pmatrix} \\ &= -3 \det \begin{pmatrix} 1 & \frac{4}{3} & \frac{2}{3} & 1 \\ 0 & 1 & 1 & 2 \\ 0 & -3 & -1 & -1 \end{pmatrix} \\ &= -3 \det \begin{pmatrix} \frac{8}{3} & \frac{16}{3} & 9 & 2 \\ 1 & 1 & 2 & 2 \\ -3 & -1 & -1 & -1 \end{pmatrix} \\ &= -3 \det \begin{pmatrix} 0 & \frac{8}{3} & \frac{11}{3} \\ 1 & 1 & 2 \\ 0 & 2 & 5 \end{pmatrix} \\ &= 3 \det \begin{pmatrix} \frac{8}{3} & \frac{11}{3} \\ 2 & 5 \end{pmatrix} \\ &= 18 \end{aligned}$$

(14)

$$\begin{aligned} \det \begin{pmatrix} 3 & -2 & 1 & -2 \\ -5 & -3 & 0 & -4 \\ 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & -2 \end{pmatrix} &= \det \begin{pmatrix} 0 & 1 & -2 & -2 \\ 0 & -8 & 5 & -4 \\ 1 & -1 & 1 & 0 \\ 0 & 2 & 0 & -2 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} &= \det \begin{pmatrix} 1 & -2 & -2 \\ -8 & 5 & -4 \\ 2 & 0 & -2 \end{pmatrix} \\ &= \det \begin{pmatrix} 1 & -2 & -2 \\ 0 & -11 & -20 \\ 0 & 4 & 2 \end{pmatrix} \\ &= \det \begin{pmatrix} -11 & -20 \\ 4 & 2 \end{pmatrix} \\ &= 58 \end{aligned}$$

(15)

$$\begin{aligned} \det \begin{pmatrix} -4 & -2 & 2 & 0 \\ 5 & 3 & 5 & 1 \\ 1 & -3 & 3 & -5 \\ 2 & 2 & -1 & 1 \end{pmatrix} &= \det \begin{pmatrix} 0 & -14 & 14 & -20 \\ 0 & 18 & -10 & 26 \\ 1 & -3 & 3 & -5 \\ 0 & 8 & -7 & 11 \end{pmatrix} \\ &= \det \begin{pmatrix} -14 & 14 & -20 \\ 18 & -10 & 26 \\ 8 & -7 & 11 \end{pmatrix} \\ &= -14 \det \begin{pmatrix} 1 & -1 & \frac{10}{7} \\ 18 & -10 & 26 \\ 8 & -7 & 11 \end{pmatrix} \\ &= -14 \det \begin{pmatrix} 1 & -1 & \frac{10}{7} \\ 0 & 8 & \frac{5}{7} \\ 0 & 1 & -\frac{3}{7} \end{pmatrix} \\ &= -14 \det \begin{pmatrix} 8 & \frac{2}{7} \\ 1 & -\frac{3}{7} \end{pmatrix} \\ &= 52 \end{aligned}$$

(16)

$$\begin{aligned} \det \begin{pmatrix} 5 & 2 & -1 & 4 \\ -4 & 1 & 3 & -2 \\ -3 & 3 & 3 & 2 \\ 4 & -2 & -4 & 0 \end{pmatrix} &= 5 \det \begin{pmatrix} 1 & \frac{2}{5} & -\frac{1}{5} & \frac{4}{5} \\ -4 & 1 & 3 & -2 \\ -3 & 3 & 3 & 2 \\ 4 & -2 & -4 & 0 \end{pmatrix} \\ &= 5 \det \begin{pmatrix} 1 & \frac{2}{5} & -\frac{1}{5} & \frac{4}{5} \\ 0 & \frac{17}{5} & \frac{11}{5} & \frac{4}{5} \\ 0 & \frac{21}{5} & \frac{17}{5} & \frac{22}{5} \\ 0 & -\frac{18}{5} & -\frac{16}{5} & -\frac{16}{5} \end{pmatrix} \\ &= 5 \det \begin{pmatrix} \frac{13}{5} & \frac{6}{5} & \frac{11}{5} & \frac{6}{5} \\ \frac{21}{5} & \frac{17}{5} & \frac{17}{5} & \frac{22}{5} \\ -\frac{18}{5} & -\frac{16}{5} & -\frac{16}{5} & -\frac{16}{5} \end{pmatrix} \\ &= 13 \det \begin{pmatrix} 1 & \frac{11}{13} & \frac{6}{13} \\ \frac{21}{5} & \frac{17}{5} & \frac{22}{5} \\ -\frac{18}{5} & -\frac{16}{5} & -\frac{16}{5} \end{pmatrix} \\ &= 13 \det \begin{pmatrix} 1 & \frac{11}{13} & \frac{6}{13} \\ 0 & -\frac{13}{13} & \frac{13}{13} \\ 0 & -\frac{2}{13} & -\frac{20}{13} \end{pmatrix} \\ &= 13 \det \begin{pmatrix} -\frac{15}{13} & \frac{32}{13} \\ -\frac{20}{13} & -\frac{13}{13} \end{pmatrix} \\ &= 28 \end{aligned}$$

(17)

$$\begin{aligned} \det \begin{pmatrix} -5 & 0 & 4 & -3 \\ -4 & -3 & -3 & -4 \\ -1 & 3 & -4 & 3 \\ -5 & 5 & 4 & 1 \end{pmatrix} &= -\det \begin{pmatrix} -5 & 0 & 4 & -3 \\ -4 & -3 & -3 & -4 \\ 1 & -3 & 4 & -3 \\ -5 & 5 & 4 & 1 \end{pmatrix} \\ &= -\det \begin{pmatrix} 0 & -15 & 24 & -18 \\ 0 & -15 & 13 & -16 \\ 1 & -3 & 4 & -3 \\ 0 & -10 & 24 & -14 \end{pmatrix} \\ &= -\det \begin{pmatrix} -15 & 24 & -18 \\ -15 & 13 & -16 \\ -10 & 24 & -14 \end{pmatrix} \\ &= 15 \det \begin{pmatrix} 1 & -\frac{8}{5} & \frac{6}{5} \\ -15 & 13 & -16 \\ -10 & 24 & -14 \end{pmatrix} \end{aligned}$$

$$\begin{aligned}
&= 15 \det \begin{pmatrix} 1 & -\frac{8}{5} & \frac{6}{5} \\ 0 & -11 & 2 \\ 0 & 8 & -2 \end{pmatrix} \\
&= 15 \det \begin{pmatrix} -11 & 2 \\ 8 & -2 \end{pmatrix} \\
&= 90
\end{aligned}$$

(18)

$$\det \begin{pmatrix} -2 & 3 & 3 & 4 \\ 3 & -5 & -2 & 0 \\ 2 & 1 & -2 & 4 \\ -3 & 5 & -1 & -5 \end{pmatrix}$$

$$\begin{aligned}
&= -2 \det \begin{pmatrix} 1 & -\frac{3}{5} & -\frac{3}{5} & -2 \\ 3 & -5 & -2 & 0 \\ 2 & 1 & -2 & 4 \\ -3 & 5 & -1 & -5 \end{pmatrix} \\
&= -2 \det \begin{pmatrix} 1 & -\frac{3}{5} & -\frac{3}{5} & -2 \\ 0 & -\frac{1}{2} & \frac{2}{5} & 6 \\ 0 & 4 & 1 & 8 \\ 0 & \frac{1}{2} & -\frac{11}{2} & -11 \end{pmatrix}
\end{aligned}$$

$$= -2 \det \begin{pmatrix} -\frac{1}{2} & \frac{5}{2} & 6 \\ 4 & 1 & 8 \\ \frac{1}{2} & -\frac{11}{2} & -11 \end{pmatrix}$$

$$= \det \begin{pmatrix} 1 & -5 & -12 \\ 4 & 1 & 8 \\ \frac{1}{2} & -\frac{11}{2} & -11 \end{pmatrix}$$

$$\begin{aligned}
&= \det \begin{pmatrix} 1 & -5 & -12 \\ 0 & 21 & 56 \\ 0 & -3 & -5 \end{pmatrix} \\
&= \det \begin{pmatrix} 21 & 56 \\ -3 & -5 \end{pmatrix} \\
&= 63
\end{aligned}$$

作成：

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