Abstract

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"Complex analytic homotopy theory for non commutative Fréchet sheaves"

For homogeneous Fréchet manifolds M of semi-Fredholm operators in submultiplicative Ψ^*- and Ψ_0-algebras $\mathcal{A}$ of the pseudodifferential analysis the Oka-Grauert isomorphism

$$\pi_n(\mathcal{H}(X,M)) \;\cong\; \pi_n(\mathcal{C}(X,M)),\; n = 0, 1, 2, \ldots$$

is extended, where π_n are the homotopy sets, X a Stein manifold or a Stein space and $\mathcal{H}$ resp. $\mathcal{C}$ denotes the space of holomorphic resp. continuous maps. The above isomorphisms are also true for closed subalgebras of the Hörmander classes $\Psi^0_{\rho,\delta}(I\!R^n)$, of the Boutet de Monvel algebra for boundary value problems and of some Ψ^*-algebras on singular spaces.

The results of Grauert (1958), Bungart (1964), Gromov (1989) and the author (1998) are extended for infinite dimensional holomorphy regions in nuclear DFN-spaces with bases (e.g. Schwartz space S' of distributions on $I\!R^n$). The essential extension is of a local type concerning the Cartan factorisation method for holomorphic matrices. Some counterexamples and open problems in connection with factorisations are presented. The Nash-Moser implicit mapping theorem gives some ideas.

The nonabelian cohomology for sheaves with values in Banach-Lie groups can be considered also for countable projective limits of Banach Lie groups and for infinite dimensional Lie groups contained in locally bounded algebras.

Some applications of the cohomological Oka principle to global properties of holomorphic semi-Fredhlm functions are given. We show the following result:
Let X be a holomorphiy region, $\mathcal{I}$ a closed twosided ideal in a submultiplicative Ψ_0-algebra $\mathcal{A}$ and $p : x \longrightarrow \mathcal{A}/\mathcal{I}$ a holomorphic map with $[p(x)]^2 = p(x) \quad (\forall\, x \in X)$. Then there exists a holomorphic lifting $\tilde{p} : X \longrightarrow \mathcal{A}$ of p with $[\tilde{p}(x)]^2 = \tilde{p}(x) \quad (\forall\, x \in X)$ if there exists a continuous projection-valued lifting $c : X \longrightarrow \mathcal{A}$ of p, $[c(x)]^2 = c(x) \quad (\forall\, x \in X)$.

Corresponding results are true for analytic homogeneous Fréchet manifolds of relative inverses in certain algebras of pseudodifferential operators.