APPROXIMATING GEODESICS VIA RANDOM POINTS

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Given a 'cost' functional F on paths γ in a domain $D \subset \mathbb{R}^d$, in the form $F(\gamma) = \int_0^1 f(\gamma(t), \dot{\gamma}(t))dt$, it is of interest to approximate its minimum cost and geodesic paths. Let $X_1, \ldots, X_n$ be points drawn independently from D according to a distribution with a density. Form a random geometric graph on the points where X_i and X_j are connected when $0 < |X_i - X_j| < \epsilon$, and the length scale $\epsilon = \epsilon_n$ vanishes at a suitable rate.

For a general class of functionals F, associated to Finsler and other distances on D, using a probabilistic form of Gamma convergence, we show that the minimum costs and geodesic paths, with respect to types of approximating discrete 'cost' functionals, built from the random geometric graph, converge almost surely in various senses to those corresponding to the continuum cost F, as the number of sample points diverges. In particular, the geodesic path convergence shown appears to be among the first results of its kind.