Curvature motion perturbed by a direction-dependent colored noise

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We study the following SPDE for $\kappa = \kappa(t, \theta)$, $\theta \in S \simeq [0, 2\pi)$:

$$\frac{\partial \kappa}{\partial t} = a(\kappa) \frac{\partial^2 \kappa}{\partial \theta^2} + b(\kappa) + h(\kappa) \circ \dot{W}^Q, \quad t > 0,\ \theta \in S, \tag{1}$$

for every $N \in \mathbb{N}$, where $a \equiv a_N$, $b \equiv b_N$, $h \equiv h_N \in C_b^\infty(S)$ satisfying $a(\kappa) \geq a_0 > 0$ (strong ellipticity) and $a(\kappa) = \kappa^2$, $b(\kappa) = \kappa^3$ and $h(\kappa) = \kappa^2$ for $N^{-1} \leq \kappa \leq N$, where $C_b^\infty(S)$ is the family of smooth functions on S having bounded derivatives of all orders and the sign $\circ$ means the product in Storatonovich sense. Let Q be a positive linear self-adjoint operator defined on $L^2(S)$ which is of trace class having the eigenvalues $\{\lambda_i^2\}_{i \in \mathbb{N}}$ and the corresponding normalized eigenfunctions $\{\alpha_i(\theta)\}_{i \in \mathbb{N}}$. Without loss of generality, we can assume that $\lambda_1 \geq \lambda_2 \geq \ldots > 0$. Note that $\mathrm{tr}Q = \sum_{i=1}^\infty \lambda_i^2 < \infty$ holds. We assume that the noise term $\dot{W}^Q$ has the form:

$$\dot{W}^Q(t, \theta) = \sum_{i=1}^\infty \lambda_i \alpha_i(\theta) \dot{w}_i(t), \tag{2}$$

where $(w_i(t))_{i \in \mathbb{N}}$ are independent one-dimensional Brownian motions. Our equation SPDE (1) is derived from the motion by mean curvature (MMC) perturbed by a direction-dependent Gaussian colored noise $\dot{W}(t, \mathbf{n})$ in two-dimensional space under the convex setting by using the Gauss map. More precisely, (1) is the cutoff version of the equation which describes the motion of a closed convex curve Γ_t in a domain $D \subset \mathbb{R}^2$ which is governed by

$$V = \kappa + \circ \dot{W}(t, \mathbf{n}(t, x)), \quad x \in \Gamma_t, \tag{3}$$

where $\mathbf{n}(t, x)$ is the inward normal vector at $x \in \Gamma_t$.

In this talk, we will give an overview for the MMC and its stochastic case and then introduce a Wong-Zakai type theorem for the SPDE (1), namely, we consider a random PDE by replacing the noise in (1) with smooth one and show the convergence of the solution in law sense. Such approximation appears in a study of the sharp interface limit for the stochastic Allen-Cahn equation.

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