Equilibrium fluctuation for a momentum-conserving chain of oscillators

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(joint work with Stefano Olla, *Université Paris-Dauphine*)

In this talk we consider an chain of anharmonic oscillators perturbed by random noises conserving the total length, momentum and energy simultaneously. We study the equilibrium fluctuation of the conserved field under the hyperbolic space-time scale $(\epsilon t, \epsilon x)$. We also present a spectral gap estimate for its generator. This is a joint work in progress with Stefano Olla in Université Paris-Dauphine.

Consider the energy functional (Hamiltonian) on $\{(p_x, q_x) \in \mathbb{R}^2\}_{x\in\mathbb{Z}}$, given by

$$\mathscr{H} = \sum_{x\in\mathbb{Z}} e_x, \quad e_x = \frac{p_x^2}{2} + V(r_x),$$

where p_x and q_x are the momenta and position of the oscillator at site x, $r_x = q_x - q_{x-1}$ and V is some potential function. We shall assume that

A1. V is smooth, non-negative, convex and $V(0) = 0$;

A2. $\delta_- \triangleq \inf_{r\in\mathbb{R}} V''(r) > 0$ and $\delta_+ \triangleq \sup_{r\in\mathbb{R}} V''(r) < \infty$.

Perturb the deterministic Hamiltonian system corresponding to $\mathscr{H}$ with certain random noise conserving the total length $\sum r_x$, total momentum $\sum p_x$ and total energy $\mathscr{H}$. The equilibrium states of the chain are canonical Gibbs measures $\pi_{\beta,\bar{p},\tau}$ associated to β (temperature inverse), $\bar{p}$ (momenta) and τ (tensor). For scaling parameter $\epsilon > 0$, consider the fluctuation field of the conserved profile $w_x = (p_x, r_x, e_x)$ under the scale $(\epsilon t, \epsilon x)$:

$$Y_\epsilon(t, dy) = \sqrt{\epsilon} \sum_{x\in\mathbb{Z}} (w_x(\epsilon^{-1}t) - E_{\pi_{\beta,\bar{p},\tau}}[w_0])\delta_{\epsilon x}(dy).$$

We show that when starting from $\pi_{\beta,\bar{p},\tau}$, $Y_\epsilon(\cdot, dy)$ converges weakly to the solution of

$$\partial_t w(t, y) = D_{\beta,\bar{p},\tau} \partial_y w(t, y), \quad w(0, y) = \sigma_{\beta,\bar{p},\tau}^{1/2} \dot{B}_y,$$

where $D_{\beta,\bar{p},\tau}$ and $\sigma_{\beta,\bar{p},\tau}$ are deterministic matrixes and $\{\dot{B}_y\}_{y\in\mathbb{R}}$ is a three-dimensional standard white noise on $\mathbb{R}$.

We also show a spectral gap of order $O(n^{-2})$ for the generator of the dynamics under an additional condition that δ_+/δ_- is close to 1. This results is useful for the study on the energy fluctuation in mechanical equilibrium.

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