Dimension-free Harnack inequalities for the stochastic partial differential equation with reflection

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Abstract

In this talk, we will mainly study the log-Harnack inequality for the transition semigroup P_t associated with the following reflected stochastic partial differential equation (SPDE) on the bounded interval $[0, 1]$ driven by a space-time white noise.

$$\begin{cases} \dfrac{\partial u}{\partial t}(t,x) = \dfrac{1}{2}\dfrac{\partial^2 u}{\partial x^2}(t,x) + b(u(t,x)) \\ \qquad\qquad + \sigma(u(t,x))\dot{W}(t,x) + \eta(dtdx),\ t > 0,\ x \in (0,1), \\ u(t,0) = u(t,1) = 0,\ t \geq 0, \\ u(0,x) = h(x),\ x \in [0,1], \\ u(t,x) \geq 0,\ t > 0,\ x \in [0,1]\ a.s., \end{cases}$$

The log-Harnack inequality is one kind of dimension-free Harnack inequality, which is initially introduced by F.-Y Wang. We show that P_t satisfies the log-Harnack inequality for each $t > 0$ based on its gradient estimate. To establish the desired gradient estimate, the penalized method for the reflected SPDE and the comparison principle for SPDEs are used. We also apply the log-Harnack inequality to the study of the strong Feller property, uniqueness of invariant measures, the entropy-cost inequality, and some estimates of the transition density of P_t with respect to its invariant measure.