On uniqueness of Dirichlet forms related to interacting systems with an infinite number of particles

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An infinite system of interacting Brownian motions in $\mathbb{R}^d$ can be represented by a stochastic process on the (unlabeled) configuration space S, the set of integer valued radon measures on $\mathbb{R}^d$. This process can be constructed by means of several probabilistic argument. Among them, Osada [2] constructed the processes in general setting by using Dirichlet form technique. For a suitable probability measure μ on S, the Dirichlet form $(\mathcal{E}^{\mathsf{upr}}, \mathcal{D}^{\mathsf{upr}})$ introduced in [2] is obtained by the smallest extension of the bilinear form $(\mathcal{E}^{\mu}, \mathcal{D}^{\mu}_{\circ})$ on $L^2(\mathsf{S}, \mu)$ with domain $\mathcal{D}^{\mu}_{\circ}$ defined by

$$\mathcal{E}^{\mu}(f,g) = \int_{\mathsf{S}} \mathbb{D}[f,g](\mathsf{s})\, \mu(d\mathsf{s}), \quad \mathbb{D}[f,g](\mathsf{s}) = \frac{1}{2}\sum_{i=1}^{\infty} \nabla_{s_i}\check{f} \cdot \nabla_{s_i}\check{g},$$

$$\mathcal{D}^{\mu}_{\circ} = \{f \in \mathcal{D}_{\circ} \cap L^2(\mathsf{S}, \mu)\, ;\, \mathcal{E}^{\mu}(f,f) < \infty\},$$

where $\mathcal{D}_{\circ}$ is the set of all local smooth functions on S, $\check{f}$ is a symmetric function such that $\check{f}(s_1, s_2, \dots) = f(\mathsf{s})$, $\cdot$ is the inner product in $\mathbb{R}^d$, and $\mathsf{s} = \sum_i \delta_{s_i}$ denotes a configuration. This Dirichlet form is the decreasing limit of the Dirichlet forms associated with finite systems of interacting Brownian motions in bounded domains $S_R = \{x \in \mathbb{R}^d; |x| \le R\}$ with a boundary condition. Because of the boundary condition, when a particle touches the boundary, it disappears. And particles enter the domain from the boundary according to the reversible measure μ.

On the other hand, Lang [1] constructed the infinite system of Brownian motions as a limit of stochastic dynamics in bounded domains S_R by taking the finite systems with another boundary condition. In his finite systems when a particle hits the boundary, it reflects and the number of particles in the domain are invariant. His process associated with the Dirichlet forms $(\mathcal{E}^{\mathsf{lwr}}, \mathcal{D}^{\mathsf{lwr}})$ the increasing limit of the Dirichlet forms associated with them.

In this talk, we discuss the relation between these Dirichlet forms $(\mathcal{E}^{\mathsf{upr}}, \mathcal{D}^{\mathsf{upr}})$ and $(\mathcal{E}^{\mathsf{lwr}}, \mathcal{D}^{\mathsf{lwr}})$. The main purpose of this paper is to give a sufficient condition for

$$(\mathcal{E}^{\mathsf{lwr}}, \mathcal{D}^{\mathsf{lwr}}) = (\mathcal{E}^{\mathsf{upr}}, \mathcal{D}^{\mathsf{upr}}).$$

References

[1] Lang, R., Unendlich-dimensionale Wienerprocesse mit Wechselwirkung I, II, Z. Wahrschverw. Gebiete **38, 39** (1977) 55-72, (1978) 277-299.

[2] Osada, H., Dirichlet form approach to infinite-dimensional Wiener processes with singular interactions, Commun. Math. Phys. **176** (1996) 117-131.