Large deviation of a tagged particle position in 1D symmetric exclusion process

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We study a fluctuation property of a tagged particle in the one-dimensional symmetric simple exclusion process (SEP) starting from the stationary measure with a uniform density $\rho, 0 < \rho < 1$. Let X_t denote the position of the tagged particle starting from the origin at time t and $\langle \cdots \rangle$ the expectation value. By the symmetry the mean position is zero, $\langle X_t \rangle = 0$, and in 1983 Arratia showed $\langle X_t^2 \rangle \sim \frac{2(1-\rho)}{\rho}\sqrt{\frac{t}{\pi}}$ as $t \to \infty$. The large deviation principle for X_t was established recently by Sethuraman and Varadhan [SV]. We will present an exact formula for the large deviation function. In fact our analysis has so far been done only for the moments. Our formula for the large deviation function is the one which is naturally expected from them.

Our results can be stated for the step initial condition with different densities, $\rho_\pm, 0 \leq \rho_\pm \leq 1$, in both directions, and can also be translated to the large deviation of the integrated current $N(x,t)$ at an arbitrary position $x \in \mathbb{Z}$. For the generating function of $N(x,t)$, we find

$$\lim_{t\to\infty} \log\langle e^{\lambda N(x,t)}\rangle/\sqrt{t} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n}\omega(\lambda)^n(\Xi_n(\xi) + \Xi_n(-\xi)) \\ - \xi(\log(1+\rho_+(e^\lambda - 1)) - \log(1+\rho_-(e^{-\lambda}-1))).$$

Here $x = -2\xi\sqrt{t}$ and

$$\omega(\lambda) = \rho_+(e^\lambda - 1) + \rho_-(e^{-\lambda}-1) + \rho_+\rho_-(e^\lambda - 1)(e^{-\lambda}-1),$$
$$\Xi_n(\xi) = \frac{1}{\sqrt{n\pi}}e^{-n\xi^2} - \xi\mathrm{erfc}(\sqrt{n}\xi),\ n \in \mathbb{Z}_+.$$

This is a generalization of a previous work on the current at the origin studied by Derrida and Gershenfeld, and also of another work on the single file diffusion of Brownian particles by Krapivsky, Mallick and Sadhu. Our approach uses recently developed techniques to study the one dimensional KPZ equation and asymmetric exclusion process, such as the Bethe ansatz, stochastic duality and nested contour formula for the deformed moments. We also discuss a fluctuation theorem like relation for the tagged particle and a connection to the macroscopic fluctuation theory (MFT).

The presentation is based on a collaboration [IMS] with T. Imamura and K. Mallick.

[SV] S. Sethuraman, S.R.S. Varadhan, Large deviations for the current and tagged particle in 1D nearest-neighbor symmetric simple exclusion, Ann. Prob. 41, 1461-1512 (2013).

[IMS] T. Imamura, K. Mallick, T. Sasamoto, Large deviations of a tracer in the symmetric exclusion process, Phys. Rev. Lett. 118, 160601 (2017).