On the asymptotics of the free energy of directed polymers in random environment in $1+1$ dimension

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We consider the free energy $F(\beta)$ of the directed polymers in random environment in $1+1$-dimension. It is known that $F(\beta)$ is of order $-\beta^4$ as $\beta \to 0$ and it has been conjectured that $\lim_{\beta\to 0} \dfrac{F(\beta)}{\beta^4} = -\dfrac{1}{24}$. In this talk, we will see that under a certain concentration condition of the environment,

$$\lim_{\beta\to 0} \frac{F(\beta)}{\beta^4} = \lim_{T\to\infty} \frac{1}{T} P_{\mathcal{Z}} \left[\log \mathcal{Z}_{\sqrt{2}}(T)\right] = -\frac{1}{6},$$

where $\{\mathcal{Z}_\beta(t,x) : t \geq 0, x \in \mathbb{R}\}$ is the unique mild solution to the stochastic heat equation

$$\frac{\partial}{\partial t}\mathcal{Z} = \frac{1}{2}\Delta \mathcal{Z} + \beta \mathcal{Z}\dot{\mathcal{W}}, \quad \lim_{t\to 0} \mathcal{Z}(t,x)dx = \delta_0(dx),$$

where $\mathcal{W}$ is a time-space white noise and

$$\mathcal{Z}_\beta(t) = \int_{\mathbb{R}} \mathcal{Z}_\beta(t,x)dx.$$