Mosco convergence for infinite particle systems related to random matrices

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The universality of random matrices (or log-gases), which is a central issue in random matrix theory, has been developed rapidly in the several decades. From these result, we obtain universal random point fields as a limit. Namely, let $\{\mu^N\}_{N\in\mathbb{N}}$ be random point fields in certain class such that μ^N has N particles, then μ^N converges universal random point field μ, which has infinitely many particles, as the number of particle N goes to infinity. The limit objects μ are, for example, the sine, Airy, and Ginibre random point field, which are logarithmic correlated infinite particle systems. We may regard the universality of random matrices as the central limit theorem for strongly correlated particle systems.

As a next issue, we would like to establish a dynamical counterpart of it, that is, we consider finite particle approximation for dynamics with infinitely many particles. Consider a distorted Brownian motion with respect to μ^N as a finite particles dynamics. Taking a limit as N goes to infinity, this finite particles dynamics must converge to a distorted Brownian motion with respect μ, and the limit must be universal dynamical object. For example, the limit dynamics are the Dyson model, Airy interacting infinite-dimensional SDE, and Ginibre interacting infinite-dimensional SDE. In this case, because each particle interact with each other by logarithmic potential, which is long-range potential, hence the limit transition is sensitive problem. Although it contains such difficulty, we establish a general theorem for finite particle approximation by Mosco convergence technique. Our result is applicable to many models, as a result, our theorem derives a dynamical universality from the universality of random matrices immediately under reasonable conditions.

This talk is based on joint work with Hirofumi Osada in Kyushu university.

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