Stochastic complex Ginzburg-Landau equation with space-time white noise

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This is a jointwork with Masato HOSHINO (Waseda University) and Nobuaki NAGANUMA. The preprint is uploaded on arXiv Preprint Server (arXiv:1702.07062).

The main objective of this talk is to prove local well-posedness of the stochastic cubic complex Ginzburg-Landau equation on the three-dimensional torus $\mathbf{T}^3 = (\mathbf{R}/\mathbf{Z})^3$ of the following form by using Gubinelli-Imkeller-Perkowski's paracontrolled calculus:

$$\partial_t u = (i+\mu)\triangle u + \nu(1-|u|^2)u + \xi, \qquad t>0, \quad x \in \mathbf{T}^3.$$

Here, $i = \sqrt{-1}$, $\mu > 0$, $\nu \in \mathbf{C}$ are constants and ξ is *complex-valued* space-time white noise, that is, a centered complex Gaussian random field with covariance

$$\mathbb{E}[\xi(t,x)\xi(s,y)] = 0, \qquad\qquad \mathbb{E}[\xi(t,x)\overline{\xi(s,y)}] = \delta(t-s)\delta(x-y),$$

where δ denotes the Dirac delta function.

We approximate ξ by a smeared noise ξ^ϵ with a parameter $0 < \epsilon < 1$ (by killing the high frequencies) so that $\xi^\epsilon \to \xi$ as $\epsilon \downarrow 0$ in an appropriate topology. We consider a renormalized equation

$$\partial_t u^\epsilon = (i+\mu)\triangle u^\epsilon + \nu(1-|u^\epsilon|^2)u^\epsilon + \nu C^\epsilon u^\epsilon + \xi^\epsilon, \qquad t>0, \quad x \in \mathbf{T}^3,$$

where C^ϵ is a suitably chosen complex constant which diverges as $\epsilon \downarrow 0$. We show that the solution u^ϵ converges to a certain non-trivial process in an appropriate topology.

Unlike regularity structure theory, there is no "bible" for paracontrolled calculus, since it has been developed gradually. For the probabilistic part, we follow Gubinelli-Perkowski's method (2017, CMP). For the deterministic part, we follow Mourrat-Weber's method (2017+, AoP).

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