

A unification of the hypercontractivity and its exponential variant of the Ornstein-Uhlenbeck semigroup

Yuu Hariya (Tohoku University)

Let γ_d be the d -dimensional standard Gaussian measure and $\{Q_t\}_{t \geq 0}$ the Ornstein-Uhlenbeck semigroup acting on $L^1(\gamma_d)$. We show that the hypercontractivity of $\{Q_t\}_{t \geq 0}$ is equivalent to the property that

$$\left\{ \int_{\mathbb{R}^d} \exp(e^{2t} Q_t f) d\gamma_d \right\}^{1/e^{2t}} \leq \int_{\mathbb{R}^d} e^f d\gamma_d,$$

which holds for any $f \in L^1(\gamma_d)$ with $e^f \in L^1(\gamma_d)$ and for any $t \geq 0$. We then derive a family of inequalities that unifies this exponential variant and the original hypercontractivity; a generalization of the Gaussian logarithmic Sobolev inequality is obtained as a corollary. If time permits, a unification of the reverse hypercontractivity and the exponential variant will also be presented.

This talk is based on the manuscript uploaded to arXiv as arXiv:1707.03163.