A DIRECTED POLYMER IN RANDOM ENVIRONMENT WITH UNBOUNDED JUMPS

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Let $(\{X_n\}_{n\in\mathbb{N}}, P)$ be the random walk on $\mathbb{Z}^d$ starting from the origin and with the transition probability

$$P(X_{n+1} = y | X_n = x) = c_1 \exp\{-c_2 |x-y|_1^\alpha\} \quad (\alpha > 0)$$

and $(\{\eta(j,x)\}_{(j,x)\in\mathbb{N}\times\mathbb{Z}^d}, Q)$ be an independent and identically distributed Bernoulli random variables with $Q(\eta(0,0) = 1) = p \in (0,1)$. We introduce the Hamiltonian $H_n^\eta(X) = \sum_{j=1}^n \eta(j, X_j)$ and define the partition functions by

$$Z_n^{\eta,\beta} = P[\exp\{\beta H_n^\eta\}] \ (\beta \in \mathbb{R}) \text{ and } Z_n^{\eta,-\infty} = P(H_n^\eta = 0),$$

where $P[\cdot]$ denotes the expectation with respect to P. It is standard to show the existence of the so-called free energy:

$$\varphi(p,\beta) = Q\text{-a.s.} \lim_{n\to\infty} \frac{1}{n} \log Z_n^{\eta,\beta} = \lim_{n\to\infty} \frac{1}{n} Q[\log Z_n^{\eta,\beta}]$$

for any $\beta \in [-\infty, \infty)$. This model was introduced as a toy model where the continuity at the (negative) zero-temperature limit holds.

Theorem 1 (Comets-Fukushima-Nakajima-Yoshida 2015, Nakajima 2017+)**.** *For any $p \in (0,1)$,*

$$\varphi(p,\beta) \xrightarrow{\beta\to-\infty} \varphi(p,-\infty).$$

If we further take $p \to 1$ limit, the free energy (after rescaling) behaves as the time constant for a directed first passage percolation. Let ω be a homogeneous Poisson point process on $\mathbb{N} \times \mathbb{R}^d$ and define

$$\mu_1 = \lim_{n\to\infty} \frac{1}{n} \inf \left\{ \sum_{k=1}^n |x_k - x_{k-1}|^\alpha : x_0 = 0, \{(k, x_k)\}_{k=1}^n \subset \omega \right\}.$$

Theorem 2 (Comets-Fukushima-Nakajima-Yoshida 2015)**.**

$$\varphi(p,-\infty) \overset{p\to1}{\sim} -c_2\mu_1(1-p)^{-\alpha/d}.$$

The above first passage percolation is a directed version of Howard-Newman's model and is interesting in its own right. In particular, it is numerically observed that there is a transition in the behavior of the minimizing path: for large α the path has only small jumps whereas for small α, it contains big jumps.

In the talk, I will briefly review the past results with backgrounds and then report some (rather primitive) mathematical results on the size of jumps.