Limit theory for geometric statistics of clustering point processes

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Let $\mathcal{P}$ be a clustering point process on $\mathbb{R}^d$ and let $\mathcal{P}_n := \mathcal{P} \cap W_n$ be its restriction to the windows $W_n \subset \mathbb{R}^d$. We consider the statistic

$$H_n^\xi := \sum_{x \in \mathcal{P}_n} \xi(x, \mathcal{P}_n)$$

where $\xi(x, \mathcal{P}_n)$ denotes a score function representing the interaction of x with respect to $\mathcal{P}_n$. When ξ depends on local data in the sense that its radius of stabilization is well controlled, we establish expectation asymptotics, variance asymptotics, and central limit theorems for H_n^ξ as well as for the random measures

$$\sum_{x \in \mathcal{P}_n} \xi(x, \mathcal{P}_n) \delta_{n^{-1/d}x},$$

as $W_n \uparrow \mathbb{R}^d$. This gives the limit theory for non-linear geometric statistics of determinantal point processes with fast decreasing kernels, including the Ginibre ensemble, extending the Gaussian fluctuation results of Soshnikov to non-linear statistics. It also gives limit theory for geometric statistics of permanental input as well as the zero set of Gaussian entire functions, extending the central limit theorems of Nazarov and Sodin, which are also confined to linear statistics. In this way we obtain limit theory for statistics of simplicial complexes, Morse critical points, germ-grain models, and random graphs whenever the input is clustering. Our approach depends on a factorial moment expansion introduced by Blaszczyszyn in 1995 for expected values of functions of general point processes.

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