MAXIMAL EDGE-TRaversal TIME IN FIRST PASSAGE PERCOLATION

SHUTA NAKAJIMA

First passage percolation (FPP) was first introduced by Hammersley and Welsh in 1965. It can be thought of as a model for the speed to percolate some material. In this talk, we focus on the maximal edge-traversal time of optimal paths in FPP and investigate the order of the growth. We shall give precise definitions below.

Let $E(z^d)$ be the set of undirected nearest-neighbor edges. We place a non-negative random variables $\tau_e$ on each edge $e$ as the passage time. Assume $\{\tau_e\}_{e \in E(z^d)}$ are i.i.d. random variables with distribution $F$. We say $\Gamma = \{x_i\}_{i=0}^{k} \subset Z^d$ is a path from $x$ to $y$ (we write $\Gamma : x \to y$) if $x_0 = x$, $x_k = y$ and $|x_i - x_{i-1}| = 1$ for $i = 1, \cdots , k$. Given a path $\Gamma = \{x_i\}_{i=0}^{k}$, the passage time of $\Gamma$ is defined as $t(\Gamma) = \sum_{i=1}^{k}\tau_{(x_{i-1}, x_i)}$ and we set first passage time $T(x, y)$ as $T(x, y) = \inf_{\Gamma : x \to y} t(\Gamma)$ for $x, y \in Z^d$. Let $Opt_n$ be the set of optimal paths from origin to $nc_1$ and $\Xi(\Gamma) = \max\{\tau_{(x_{i-1}, x_i)} : 1 \leq i \leq k\}$ for $\Gamma = \{x_i\}_{i=0}^{k} \in Opt_n$.

Let $F$ be the infimum of the support of $F$ and $\mu(d)$, $\bar{\mu}(d)$ the critical probability of d-dim percolation, oriented percolation model, respectively. Then $F$ is said to be useful if either holds:

(i) $F = 0$, $F(\{0\}) < \mu(d)$, (ii) $F \geq 0$, $F(\{\bar{F}\}) < \bar{\mu}(d)$.

It is easy to check that if $F$ is useful, $Opt_n$ is not empty almost surely. It is known from the result of van den Berg and Kesten in [1] that if $F$ is unbounded and useful,

$$\min_{\Gamma \in Opt_n} \Xi(\Gamma) \to \infty \ a.s.$$ 

Our purpose is to investigate the actual growth of the order of $\Xi(\text{Opt}_n)$.

**Theorem 1.** Suppose $d \geq 2$, $F$ is useful, and there exist $a > 1$, $c_1 - c_4$, $t_1$, $r > 0$ such that for any $t \geq t_1$, $c_1 e^{-ct^r} \leq F([t, at]) \leq c_3 e^{-ct^r}$. Then, there exists $K > 0$ such that,

$$\mathbb{P}\left(K^{-1} f_{d,r}(n) \leq \min_{\Gamma \in \text{Opt}_n} \Xi(\Gamma) \leq \max_{\Gamma \in \text{Opt}_n} \Xi(\Gamma) \right) \to 1,$$

where, we set

$$f_{d,r}(n) := \begin{cases} 
(\log n)^{\frac{1}{2}} & \text{if } 0 < r < d - 1 \\
(\log n)^{\frac{1}{2}} (\log \log n)^{\frac{1}{2r}} & \text{if } r = d - 1 \\
(\log n)^{\frac{1}{2}} (\log \log n)^{\frac{1}{2}} & \text{if } d - 1 < r < d \\
(\log n)^{\frac{1}{2}} (\log \log n)^{-\frac{1}{2}} & \text{if } r = d \\
(\log n)^{\frac{1}{2}} & \text{if } d < r.
\end{cases}$$

**Theorem 2.** Suppose $d \geq 2$, $F$ is useful, $\mathbb{E}[\tau^4] < \infty$ and there exist $0 < \alpha$, $c$, $t_1$ and $a > 1$ such that for any $t \geq t_1$, $F([t, at]) \geq c t^{-\alpha}$. Then, there exists $K > 0$ such that,

$$\mathbb{P}\left(K^{-1} \frac{\log n}{\log \log n} \leq \min_{\Gamma \in \text{Opt}_n} \Xi(\Gamma) \leq \max_{\Gamma \in \text{Opt}_n} \Xi(\Gamma) \right) \to 1.$$ 

**References**


(Shuta Nakajima) RESEARCH INSTITUTE IN MATHEMATICAL SCIENCES, KYOTO UNIVERSITY, KYOTO, JAPAN

E-mail address: njima@kurims.kyoto-u.ac.jp