土譲汚染の数理モデル
-数学・工学・産業の異分野連携のプラットフォーム創出を目指して-

Mathematical Modeling for Anomalous Diffusion in Soil
-Creating an Interdisciplinary Platform for Taking Aim at Mathematical Innovation-

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Nippon Steel knows that mathematics is a very powerful language that can describe the essence of problems and has been creating an interdisciplinary platform for taking aim at technological innovation based on mathematics. For this platform, mathematicians and engineers have come together, contemplating social problems and taking voluntary actions.

The scientific topic is the issue of anomalous diffusion in soil. The approach is highly diverse, with a boundary that encompasses mathematics, engineering, and industry. The ultimate objective is to determine the microstructure of soil through averaged spatial data analysis, such as contaminant concentration, and to predict the progress of soil contamination.

Figure 1 shows that it is often the case with mass diffusion in a porous medium such as soil that the numerical simulations using traditional advection diffusion equations fail to predict the observation results of a real phenomenon observed in the field or in laboratory tests. The numerical experiments using CTRW says that the mean squared displacement of particles grows in proportion to the fractional power of time [1].

Figure 2 shows that CTRW is linked with the fractional order PDE in terms of time [2]. This means that anomalous diffusion depends on the degree of history to be retained from the initial time to the current time. The smaller α is, the more history will be retained. We can combine the physical meaning of alpha (that stems from possible obstacles that delay the particle's jump) with the mathematical reasoning.

Thus, how do we combine the microstructure with the mechanism for determining the value? What are the geometric invariants? How do we combine the geometric invariants with the PDE in a mathematical framework? These are our next targets. The current progress of our discussion will be presented using the following methods.

1. Analytical description for mathematically explaining the facts discovered by the experiments (a macro-scale viewpoint)
2. Characterization of the geometric features of the specimens of a 3D CT-image (a micro-scale viewpoint)
3. Deductive reasoning to derive a fractional differential equation using the homogenization method (a multi-scale viewpoint)

REFERENCES

[1]Y. Hatano and N. Hatano, Dispersive transport of ions in column experiments: An explanation of long-tailed profiles, Water Resources Research, 34, 1027-1033 (1998)
[2] J. Nakagawa, K. Sakamoto, M. Yamamoto, Overview to mathematical analysis for fractional diffusion equations-new mathematical aspects motivated by industrial collaboration, Journal for Math-for-industry, Vol.1 (2009B-9), 00.157-163

Analysis for Anomalous Diffusion in Laboratory Experiments

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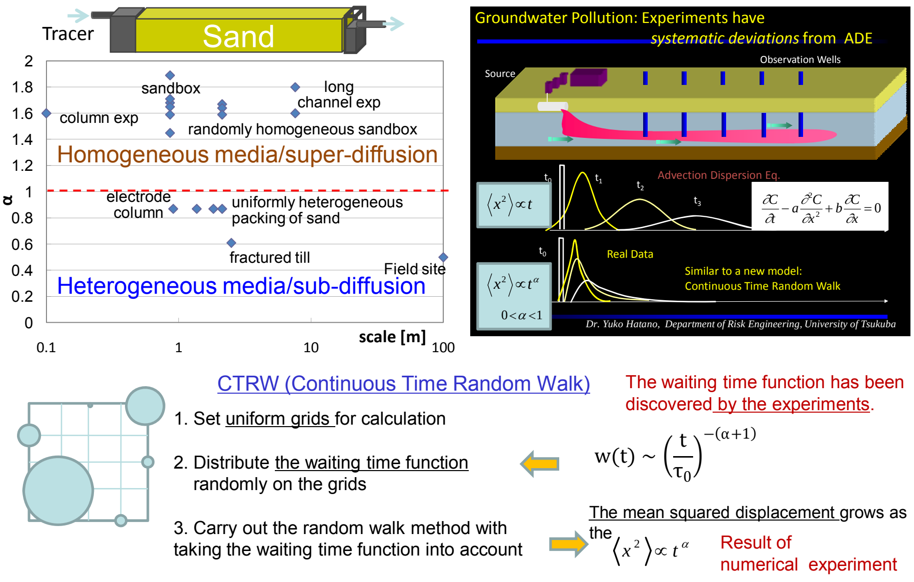

Fig.1 Prediction of soil contamination

An Approach for Inverse Problems from a Multi-scale Viewpoint Utilizing a Combination of Stochastic, Analytic, and Geometric Modeling

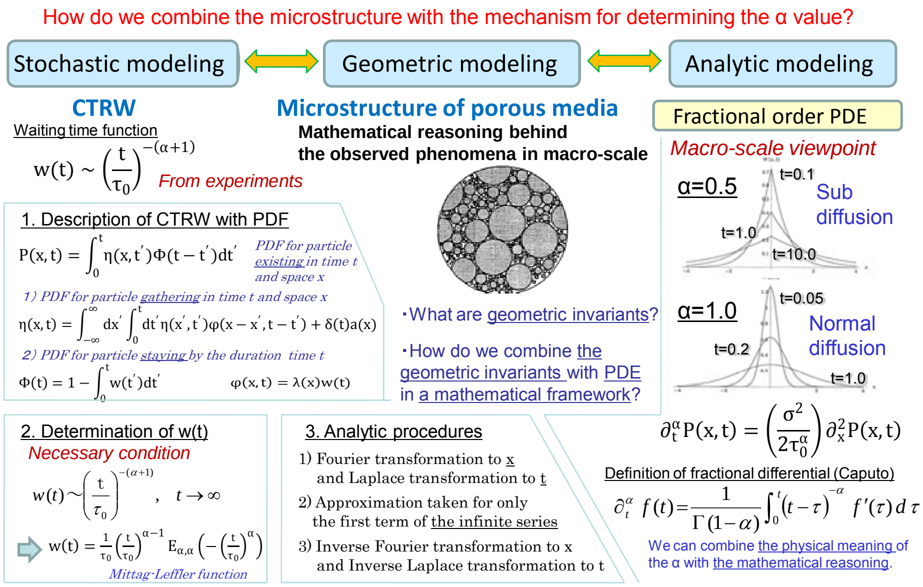

Fig.2 Outline of scientific topic