Low dimensional topology and number theory XVII

IMI Auditorium Rm 413, West 1st Bldg. D, Kyushu University (Ito Campus)
16th March, 2026 $\sim$ 19th March, 2026

Program

March 16th (Mon)

14:00 – 15:00
Tatsuya Ohshita (Gunma University)
Asymptotic behavior of ideal class groups along Galois representations and its topological analogue

15:20 – 16:20
Honami Sakamoto (Ochanomizu University)
Linear recurrent sequences in arithmetic topology

March 17th (Tues)

10:00 – 11:00
Takenori Kataoka (Tokyo University of Science)
Kida's formula for graphs with ramifications

11:20 – 12:20
Sohei Tateno (Nagoya University)
An analogue of non-commutative Iwasawa theory for fibered knots

14:00 – 15:00
Manabu Ozaki (Waseda University)
Reconstruction of number fields from locally cyclotomic fundamental groups

15:20 – 16:20
Bora Yalkinoglu (IRMA - Université de Strasbourg)
Towards a geometrization of Shintani invariants

March 18th (Wednes)

10:00 – 11:00
Kohei Kitamura (Osaka University)
Multiple polylogarithms at non-positive indices and combinatorics of Magnus polynomials

11:20 – 12:20
Noriyuki Otsubo (Chiba University)
Abelian coverings of the projective line minus four points and the adelic Gaussian hypergeometric function

14:00 – 15:00
Toshiki Matsusaka (Kyushu University)
On the Alexander polynomials of modular knots

15:20 – 16:20
Hiroyuki Ogawa (Osaka University)
A Poincare-convex set associated with a linear operator on a real Hilbert space

March 19th（Thurs)

10:00 – 11:00
Dohyeong Kim (Seoul National University)
On the cohomological approach to trilinear arithmetic symbols

11:20 – 12:20
Atsuki Kuramoto (Kyushu University)
On triple quadratic residue symbols in real quadraic fields

Abstract

• Tatsuya Ohshita (Gunma University)

Asymptotic behavior of ideal class groups along Galois representations and its topological analogue

Let T be an integral p-adic representation of the absolute Galois group of a number field. In this talk, by using the Selmer group of T, we shall describe the asymptotic behavior of certain quotients of ideal class groups along the non-commutative tower of fields arising from T (partially based on a joint work with T. Hiranouchi). We shall also note the analogous results in the arithmetic topology.

• Honami Sakamoto (Ochanomizu University)

Linear recurrent sequences in arithmetic topology

The class numbers (the size of H_1) of branched covers of knots are computed using the cyclic resultants of the Alexander polynomials. In the spirit of arithmetic topology, we discuss a theorem, observations, and a conjecture on double twist knots, genus two two-bridge knots, and Pretzel knots, asserting that "If a prime number p divides the class number of some odd-th cyclic cover of a knot K, then K admits a liminal $\mathrm{SL}_2\mathbb{Z}_p$-character, namely, a reducible character that may be arbitrarily approximated by irreducible characters." We also discuss a finite analogue $\mathcal{P}^0_{\mathcal{A}}$ of the ring of algebraic numbers, characterized by J. Rosen using linear recurrent sequences, within "the poor man's adele ring" $\mathcal{A} = \prod_p \mathbb{Z}_p / \bigoplus_p \mathbb{Z}/p\mathbb{Z}$. We present constructions of non-trivial elements, results on the densities of the zero sets of such recurrent sequences, positive-characteristic analogues, and a perspective towards a possible knot-theoretic counterpart. This talk is based on several joint works with D. Matsuzuki, R. Tange, and J. Ueki.

• Takenori Kataoka (Tokyo University of Science)

Kida's formula for graphs with ramifications

Recently Iwasawa theory for graphs is being developed. A significant achievement includes an analogue of Iwasawa's class number formula, which describes the asymptotic growth of the numbers of spanning trees for $\mathbb{Z}_p$-coverings of graphs. Moreover, an analogue of Kida's formula concerning the behavior of the λ- and μ-invariants is obtained for unramified coverings. In this talk, we present Kida's formula for possibly ramified coverings.

• Sohei Tateno (Nagoya University)

An analogue of non-commutative Iwasawa theory for fibered knots

It is known that there are deep analogies between low-dimensional topology and algebraic number theory. In particular, Alexander-Fox theory is considered to be the topological counterpart of Iwasawa theory. In this talk, based on the insights from non-commutative Iwasawa theory, we construct a knot invariant for fibered knots, which is a refinement of classical Alexander polynomials. As an application, we generalize Fox's classical formula for the sizes of first homology groups, extending it from cyclic covers to certain non-abelian covers of S^3 branched along knots. We further explore several consequences of this formula, together with numerical examples.

• Manabu Ozaki (Waseda University)

Reconstruction of number fields from locally cyclotomic fundamental groups

We introduce the locally cyclotomic fundamental group of a number field (or the spectrum of its integer ring), which serves as the "right" number field analogue of the etale fundamental group of curves over finite fields. We then present various anabelian results for this fundamental group. Specifically, we provide a Neukirch-Uchida type theorem and its m-step solvable variant for locally cyclotomic fundamental groups. In particular, we demonstrate that the maximal meta-abelian quotient of a locally cyclotomic fundamental group determines the number field. Finally, we provide a mono-anabelian reconstruction of a number field from its locally cyclotomic fundamental group.

• Bora Yalkinoglu (IRMA - Université de Strasbourg)

Towards a geometrization of Shintani invariants

Shintani invariants were introduced by Shintani in 1976 and are conjectured to generate abelian extensions of real quadratic number fields, thereby giving a conjectural solution to Hilbert's twelfth problem in this setting. Shintani famously defined these invariants using the double sine function, but despite many efforts, their algebraicity remains mysterious. A major obstacle is that the double sine function originates in quantum integrable systems (as an eigenfunction of a quantum Toda Hamiltonian), and it is not clear how to approach it from a number-theoretic or geometric perspective. Very recently, we found a new bridge between quantum and classical Toda systems by passing through quantum Toda systems at roots of unity; in particular, we use the cyclic quantum dilogarithm to express Shintani invariants. In this talk, we explain this approach and show how Shintani invariants arise naturally in the geometric framework of classical Toda systems. Time permitting, we also present a curious analogy with the volume conjecture in knot theory.

- Kohei Kitamura (Osaka Universsity)

Multiple polylogarithms at non-positive indices and combinatorics of Magnus polynomials

In this talk I present a combinatorial and algebraic approach to multiple polylogarithms with non-positive multi-indices. Using Magnus polynomials in a free associative algebra, I give an explicit Magnus-type product representation for products of mono-indexed non-positive MPLs and derive new $\mathbb{Q}$-linear functional equations among these functions arising from permutations and algebraic relations of the Magnus polynomials.

- Noriyuki Otsubo (Chiba University)

Abelian coverings of the projective line minus four points and the adelic Gaussian hypergeometric function

We consider a tower of hypergeometric curves, which constitute abelian coverings of the projective line minus four points. Under a freeness conjecture, this gives a Galois representation with values in GL_2 of a large group ring with "three variables". Its trace is called the adelic Gaussian hypergeometric function and interpolates all the Gaussian hypergeometric functions over all finite fields. This generalizes the work of Ihara and Anderson on the tower of Fermat curves, GL_1 of a group ring with "two variables" and the adelic beta function.

- Toshiki Matsusaka (Kyushu University)

On the Alexander polynomials of modular knots

Closed geodesics associated with indefinite binary quadratic forms, or equivalently with real quadratic irrationals, have long been studied as geometric $\mathrm{SL}_2(\mathbb{Z})$-invariant. Building on the Birman-Williams approach to Lorenz knots and following the notion of modular knots introduced by Ghys, this talk investigates the topological $\mathrm{SL}_2(\mathbb{Z})$-invariants arising from modular knots. (This is joint work with Soon-Yi Kang and Kyungbae Park).

• Hiroyuki Ogawa (Osaka University)

A Poincaré-convex set associated with a linear operator on a real Hilbert space

The numerical range is a classical visualization tool for linear operators. It is a convex subset of the complex plane that contains the spectrum, and it has been extensively studied in linear algebra and functional analysis. In 2016, R. Hannah, E. K. Ryu and W. Yin introduced the Scaled Relative Graph (SRG) as a visual analysis tool in mathematical optimization (published in 2021). Since around 2024 a number of related results have been reported. Although the SRG was developed in a framework that includes nonlinear operators, the linear case was studied by R. Pates (2021), who showed that the SRG of a linear operator is symmetric with respect to the real axis, contains the spectrum, and is Poincaré convex on each half-plane; however, the full SRG is generally neither convex nor connected. In 2024, the present author defined an essentially equivalent set by restricting attention to the upper half-plane of the complex plane. This set, called a leaf due to its shape, is a Poincaré convex set containing the spectrum with nonnegative imaginary part. While the numerical range typically exhibits a rounded shape characteristic of convex sets, the SRG (leaf) displays sharper boundary and endpoint behavior. This talk presents the definition of the SRG (leaf), fundamental properties (spectral inclusion, Poincaré convexity, its relation to the operator norm), and several applications.

• Dohyeong Kim (Seoul National University)

On the cohomological approach to trilinear arithmetic symbols

The history of trilinear arithmetic symbols started with Redei, whose construction is interpretable as a trilinear extension of the Legendre symbol. Morishita revealed its meaning in the context of arithmetic topology via arithmetic link groups, followed by his continued work with collaborators. On the other hand, its cohomological aspects are highlighted in Corsman's thesis. The speaker will explain how one can define such trilinear arithmetic symbols over number fields with enough roots of unity using cohomological techniques. The talk is based on a joint work with Morishita.

• Atsuki Kuramoto (Kyushu University)

On triple quadratic residue symbols in real quadraic fields

In this talk, I introduce triple quadratic residue symbols for certain primes in a real quadratic field k with narrow class number one. Our symbol may be regarded as a triple generalization of the quadratic residue symbol in k and also an extension of the Rédei symbol in the rationals for k. From the viewpoint of arithmetic topology, our symbol may be regarded as an arithmetic analogue of Milnor–Turaev's triple linking number of knots in a homology 3-sphere. For the construction, we determine the group presentation of the pro-2 maximal Galois group over k with restricted ramification, from which we derive our triple symbols by using mod 2 Magnus expansion. We then show that our triple symbol describes the decomposition law of a prime in a dihedral extension of degree 8 over k with restricted ramification, called Rédei type extension, and give concrete numerical examples. We also give an interpretation of our symbols in terms of Massey products in Galois cohomology. Finally, we show that our symbol agrees with the symbol derived by a different cohomological approach, which was introduced by Kim–Morishita.