Low dimensional topology and number theory XVI
In honor of Professor Hiroaki Nakamura's 60th birthday

March 25 - 28, 2025
Room E404, Building E, School of Science, Osaka University, JAPAN

Program

March 25 (Tuesday)

10:00∼11:00:
Akio Tamagawa (RIMS, Kyoto University)
Developments of anabelian geometry and their origin

11:20∼12:20
Mohamed Saïdi (University of Exeter)
On the arithmeticity of the mapping class group

14:20∼15:20
Seidai Yasuda (Hokkaido University)
Additions and M-operations

15:40∼16:40
Jakob Stix (Goethe Universität Frankfurt Am Main)
A π_1-obstruction to lifting to characteristic 0

March 26 (Wednesday)

10:00∼11:00
Pierre Dèbes (Université Lille)
Patching and Hurwitz spaces in Inverse Galois theory

11:20∼12:20
Takeshi Ogasawara (Dokkyo Medical University)
Searching for PGL(2,7) number fields with a single ramified prime

14:20∼15:20
Eiko Kin (Osaka University)
A study of braids derived from choreographic solutions of the planar N-body problem

15:40∼16:40
Hiroshi Tsunogai (Sophia University)
Reduction theory for round-version of continued fraction expansion

March 27 (Thursday)

8:00∼9:00
Florian Pop (University of Pennsylvania) (Online)
Generalizations and refinements of the t-birational Section Conjecture

11:00∼12:00
Ken Ono (University of Virginia)
Eichler-Selberg relations in complex multiplication

13:30∼14:30
Takuya Sakasai (The University of Tokyo)
Computations on Johnson cokernels

14:50∼15:50
Pierre Lochak (Université Paris Pierre et Marie Curie)
The Grothendieck-Teichmüller group: past and present

16:10∼17:10
Hiroaki Nakamura (Osaka University)
Breeding Eisenstein invariants in the profinite braid tower

About 17:20
Group Photo

18:30∼
Party

March 28 (Friday)

10:00∼11:00
Leila Schneps (Institut de Mathématiques de Jussieu)
Relationship between the double shuffle and Kashiwara-Vergne Lie algebras revisited

11:20∼12:20
Hidekazu Furusho (Nagoya University)
Kontsevich's eye, Lie graphs and associators

14:20∼15:20
Densuke Shiraishi (Tokyo University of Science)
On the ℓ-adic Galois analogue of multiple zeta values and polylogarithms

15:40∼16:40
Takao Satoh (Tokyo University of Science)
On the Johnson homomorphisms of the automorphism groups of free groups

Abstract

Pierre Dèbes (Université Lille)
Patching and Hurwitz spaces in Inverse Galois theory

Hurwitz spaces are some of the few tools that have paved the way to the development of modern inverse Galois theory. They are originally and essentially topological but have been used to produce results about problems that are arithmetic by nature. Patching is another topologically inspired technique that has parallelly led to related results in the same territory. I will explain how these two trends have appeared, developed side by side and sometimes met and united.

Hidekazu Furusho (Nagoya University)
Kontsevich's eye, Lie graphs and associators

In this talk, I will review the notion of Kontsevich's eye and the weights associated with Lie graphs. I will then discuss the associator constructed by Alekseev and Torossian.

Eiko Kin (Osaka University)
A study of braids derived from choreographic solutions of the planar N-body problem

Periodic solutions of the planar N-body problem play a significant role in the study of celestial mechanics. We use braid types as a classification tool for periodic solutions. According to the Nielsen-Thurston classification of surface automorphisms, braids can be classified into three types: periodic, reducible, and pseudo-Anosov. For a braid of pseudo-Anosov type, there is an associated stretch factor greater than 1, which is a conjugacy invariant of the braid. A simple choreography in the N-body problem is a periodic solution in which all masses chase each other along a closed loop. For each $N \geq 3$ Guowei Yu proved the existence of a family of simple choreographies consisting of $2^{N-3} + 2^{\lfloor (N-3)/2 \rfloor}$ distinct solutions. We prove that braids derived from the Yu's solutions are of pseudo-Anosov types, except in the special case where all masses move along a circle. We also identify the simple choreography whose braid type has the largest and smallest stretch factors, respectively. This is joint work with Yuika Kajihara and Mitsuru Shibayama.

Pierre Lochak (Université Paris Pierre et Marie Curie)
The Grothendieck-Teichmüller group : past and present

After reviewing the history of that group, starting from his inception in

Grothendieck's Esquisse d'un programme, I will present a somewhat novel viewpoint on this object, based on quite recent published results.

Hiroaki Nakamura (Osaka University)
Breeding Eisenstein invariants in the profinite braid tower

The monodromy action on the meta-abelian fundamental group of a once-punctured elliptic curve provides a certain family of quantities "Eisenstein invariants". In this talk, we discuss their extensions to those quantities on what B.Enriquez (2014) introduced as the elliptic Grothendieck-Teichmüller group $\widehat{GT}_{ell}$. We also provide a snapshot of $\widehat{GT}_{ell}$ with the profinite braid tower in the background from a viewpoint of my collaboration with A.Minamide (2022).

Takeshi Ogasawara (Dokkyo Medical University)
Searching for PGL(2,7) number fields with a single ramified prime

We found many PGL(2,7) number fields with a single ramified prime. I will talk about some methods for it, including computations of mod 7 modular forms of weight one and a study on ramification properties of such kind of PGL(2,7) number fields. This is a joint work with George J. Schaeffer.

Ken Ono (University of Virginia)
Eichler-Selberg relations in complex multiplication

The Eichler-Selberg trace formula expresses the trace of Hecke operators on spaces of cusp forms as weighted sums of Hurwitz-Kronecker class numbers. We extend this formula to a natural class of relations for traces of singular moduli, where one views class numbers as traces of the constant function $j_0(\tau) = 1$. More generally, we consider the singular moduli for Zagier's Hecke system of modular functions $j_m(\tau)$. For each $\nu \geq 0$ and $m \geq 1$, we obtain an Eichler-Selberg relation. For $\nu = 0$ and $m \in \{1, 2\}$, these relations are Kaneko's celebrated singular moduli formulas for the coefficients of $j(\tau)$. For each $\nu \geq 1$ and $m \geq 1$, we obtain a new Eichler-Selberg trace formula for the Hecke action on the space of weight $2\nu + 2$ cusp forms, where the traces of $j_m(\tau)$ singular moduli replace Hurwitz-Kronecker class numbers. These formulas involve a new term that is assembled from values of symmetrized shifted convolution L-functions. This is joint work with Yuqi Deng and Toshiki Matsusaka.

Florian Pop (University of Pennsylvania)
Generalizations and refinements of the t-birational Section Conjecture

After a brief introduction to Grothendieck's section conjecture (SC) and its variants, I will recall the t-birational SC over fields k of finite type over $\mathbb{Q}$, as studied and completely resolved by Bresciani. The theme of my talk will be give generalizations and "minimalistic" refinements of the classical t-birational SC, by allowing quite general base fields k, beyond the context of Grothendieck's SC.

Mohamed Saïdi (University of Exeter)
On the arithmeticity of the mapping class group

In my talk, I will report on a new result in combinatorial anabelian geometry related to the arithmetic version of the mapping class group, and a combinatorial group-theoretic approach to understand its structure. This result draws a picture that brings combinatorial anabelian geometry, anabelian geometry, and Grothendieck-Teichmüller theory closer to each other, three topics that inspired the research work of Hiroaki Nakamura throughout his entire career. This is a joint (still ongoing) research project with Shinichi Mochizuki and Shota Tsujimura from RIMS (Kyoto).

Takuya Sakasai (The University of Tokyo)
Computations on Johnson cokernels

The Johnson homomorphisms are fundamental tools for the algebraic study of the mapping class group of a surface. Determinig the cokernel has been an important problem and the Enomoto-Satoh obstruction (trace) provides a major part of the cokernel. In this talk, we discuss those Johnson cokernels that cannot be detected by the Enomoto-Satoh obstruction. This is a joint work with Shigeyuki Morita and Masaaki Suzuki.

Takao Satoh (Tokyo University of Science)
On the Johnson homomorphisms of the automorphism groups of free groups

I will talk about the Johnson homomorphisms from a viewpoint of the study of the automorphism groups of free groups. In the first half, we will give a survey for the study of the cokernels of the Johnson homomorphisms and twisted cohomology groups of the automorphism groups of free groups. In the second half, we are going to talk about the Andreadakis conjecture and related recent results.

Leila Schneps (Institut de Mathématiques de Jussieu)
Relationship between the double shuffle and Kashiwara-Vergne Lie algebras revisited

In 2012 I gave a proof that the double shuffle Lie algebra ds injects into the Kashiwara-Vergne Lie algebra krv, based on a theorem stated by Jean Écalle, and given with a sketch of proof. However, it turned out over the ensuing years that no one was able to fully complete the proof of Écalle's statement, and so the injection map from ds to krv remained conjectural. In this talk I will give a new approach which does not use Ecalle's theorem.

Densuke Shiraishi (Tokyo University of Science)
On the ℓ-adic Galois analogue of multiple zeta values and polylogarithms

The ℓ-adic Galois analogues of multiple zeta values and polylogarithms are defined using noncommutative 1-cocycles arising from the Galois action on the pro-ℓ etale fundamental groupoid of $\mathbb{P}^1\backslash\{0,1,\infty\}$ with rational base points. These objects were first introduced by Wojtkowiak and are closely related to the Soule character, along with its polylogarithmic refinement – the generalized Soule character – formulated by Nakamura-Wojtkowiak. In this talk, we will discuss the relations among ℓ-adic Galois multiple zeta values and functional equations of ℓ-adic Galois polylogarithms, building upon the foundational works of Nakamura-Wojtkowiak and also presenting the speaker's own results.

Jakob Stix (Goethe Universität Frankfurt Am Main)
A π_1-obstruction to lifting to characteristic 0

Lifting a variety from char $p > 0$ to characteristic 0 links its étale fundamental group to a discrete finitely presented group form the 2-skeleton of a triangulation of the space of complex points. We will explain the notion of a p'-discretely finitely generated pro-finite group. The question of whether the étale fundamental group of a variety in characteristic $p > 0$ admits such a "discrete structure" is related to the possibility of lifts to characteristic 0. The Roquette curve with an automorphism group exceeding the Hurwitz bound allows to construct examples with a π_1-obstructed lifting problem. (This is joint work with Hélène Esnault and Vasudevan Srinivas.)

Akio Tamagawa (RIMS, Kyoto University)
Developments of anabelian geometry and their origin

In this talk, I will try to show that we can find the origin of many (or even almost all) recent important developments of anabelian geometry in Hiroaki Nakamura's work in the 1990s. The topics will include: absolute anabelian geometry; pro-l anabelian geometry; m-step solvable anabelian geometry; combinatorial anabelian geometry; higher-dimensional anabelian geometry; and so on.

Hiroshi Tsunogai (Sophia University)
Reduction theory for round-version of continued fraction expansion

The study of reduction theory for quadratic forms has its origin in "Disquisitiones Arithmeticae" by Gauss. It can be also regarded as that for quadratic irrational numbers. In particular, for real quadratic numbers, it is closely related to continued fraction expantion (CFE) of those numbers. In this talk, we discuss on a variant of CFE of real numbers — round-version of continued fraction expantion (r-CFE) —, where we take the integer part of a real number by rounding off. We want to investigate the condition that the r-CFE of a real quadratic number is purely periodic, which seems to be related to the golden ratio. This is an on-going research joint with Kyosuke Wakaiki (Sophia University).

Seidai Yasuda (Hokkaido University)
Additions and M-operations

I hope that I have a good understanding of the importance of rings and fields. However, the addition is too complicated for the people in the world of multiplicative monoids. As a trial to reduce the complexity, I would like to introduce the notion of M-operation. After introducing my motivation, I will present some observations related to this notion through some special examples. Furthermore, I will propose analogies, and some conjectures.