Low dimensional topology and number theory XV

C502 Lecture Rm, 5F West 1st Bd, Kyushu University (Ito Campus)
5th March, 2024 $\sim$ 8th March, 2024

Program

March 5th (Tues)
10:00 – 10:25
Ryoto Tange（Waseda University)
Survey on Iwasawa theory for representations of knot groups

10:35 – 11:00
Mishty Ray (University of Calgary)
Introduction to Geometry of local Arthur packets and Vogan's conjecture

11:20 – 12:10
Sohei Tateno（Nagoya University）
On Kida's formula for $\mathbb{Z}_p^d$-covers of graphs

13:40 – 14:30
Mahiro Atsuta（Tsuda University）
Euler system for Tate motives over a totally real field

14:50 – 15:40
Anwesh Ray（Chennai Mathematical Institute)
Topological Iwasawa invariants and arithmetic statistics

March 6th (Wednes)
9:40 – 10:30
Xin Ren （Kansai University）
q-deformed rational numbers, q-deformed Farey sum and a triangulated category of A_2 quiver

10:50 – 11:40
Mai Katada (Kyushu University)
The first homology of the IA-automorphism groups of free groups with coefficients in spaces of Jacobi diagrams

13:20 – 14:10
Benjamin Collas（RIMS, Kyoto University)
Oda's prediction, a motivating conjecture

14:40 – 15:30
Séverin Philip (RIMS, Kyoto University)
The maximal degeneration method for special loci

15:50 – 16:40
Yasuhiro Wakabayashi（Osaka University）
Topological quantum field theory for differential equations in characteristic p^N

March 7th (Thurs)
9:40 – 10:30
Nadav Gropper (University of Pennsylvania/University of Haifa)
TQFTs for pro-p Poincaré duality groups

10:50 – 11:15
Yuki Ishida（Kyushu University）
The density of Borromean primes

11:25 – 11:50
Yuqi Deng (Kyushu University)
Arithmetic Dijkgraaf–Witten invariants for real quadratic fields, quadratic residue graphs, and density formulas

13:20 – 14:10
Toshiki Matsusaka (Kyushu University)
Introduction to quantum modular forms and discontinuity property of Habiro series

14:40 – 15:30
Hisatoshi Kodani (IMI, Kyushu University)
Quantum master equation and Hodge correlators

15:50 – 16:40
Soichiro Uemura (Kavli IPMU, The University of Tokyo)

A proof of the Teichmüller TQFT volume conjecture for the knot 7_3

March 8th（Fri)
10:00 – 10:50
Jun Ueki（Ochanomizu University）
Arithmetic topology and the profinite rigidity of the multivariable Alexander polynomials of links

11:10 – 12:00
Hiraku Nozawa (Ritsumeikan University)
Harmonic measures and rigidity for the Fuchsian actions of surface groups on the circle

Abstract

• Ryoto Tange (Waseda University)

Survey on Iwasawa theory for representations of knot groups

In the spirit of arithmetic topology, I will survey a knot theoretic analogue of Iwasawa theory for Galois representations. I especially focus on the L-function associated to the twisted knot module for the universal deformation of a representation of a knot group, which may be seen as an analogue of the algebraic p-adic L-function associated to the Selmer module for the universal deformation of a Galois representation. This is joint work with Takahiro Kitayama, Masanori Morishita, Yuji Terashima, Leo Benard, Anh Tran, and Jun Ueki.

• Mishty Ray (University of Calgary)

Introduction to Geometry of local Arthur packets and Vogan's conjecture Local

Arthur packets, or A-packets, are sets of representations of p-adic groups that help us realize an important classes of automorphic forms. Vogan's geometric perspective on the Langlands correspondence establishes a bijection between equivalence classes of smooth irreducible representations of $G(F)$ and simple equivariant perverse sheaves on a moduli space of Langlands parameters. This gives us the notion of an ABV-packet, which is also a set of smooth irreducible representations of $G(F)$, but now attached to any Langlands parameter. Conjecturally, ABV-packets generalize A-packets; we call this Vogan's conjecture. In recent work joint with Clifton Cunningham, we prove this conjecture for p-adic GL_n. In this talk, we will explore the geometry of the moduli space of Langlands parameters for GL_n via examples. We will summarize basic results towards Vogan's conjecture and discuss other interesting problems in this area.

• Sohei Tateno (Nagoya University)

On Kida's formula for $\mathbb{Z}_p^d$-covers of graphs

Riemann–Hurwitz formula for $\mathbb{Z}_p$-extensions over number fields is called Kida's formula, which describes the relations among Iwasawa λ-invariants, extension degrees, and ramification indices. Recently, Ray and Vallières have established Kida's formula for $\mathbb{Z}_p$-covers of graphs. In this talk, by using generalized Iwasawa μ and λ-invariants introduced by Cuoco and Monsky, we give Kida's formula for $\mathbb{Z}_p^d$-covers of graphs. We also provide several examples. This is a joint work with Kosuke Mizuno.

• Mahiro Atsuta (Tsuda University)

Euler system for Tate motives over a totally real field

In recently work, Burns-Sano constructed a higher rank Euler system for $\mathbb{Z}_p(1)$ over a totally real field that interpolates the special values of the Dedekind zeta function. In this talk, we discuss a refinement of their work. More precisely, we introduce the existence of an Euler system which interpolates more values of L-functions. This is a joint work with Naoto Dainobu and Takenori Kataoka.

• Anwesh Ray (Chennai Math. Inst.)

Topological Iwasawa invariants and arithmetic statistics

There are interesting analogies between number theory and topology. Such analogies were systematically observed by Mazur who observed parallels between the arithmetic of primes in number rings and the geometry of knots embedded in 3 manifolds. With respect to this analogy, we may formulate certain questions in arithmetic statistics and then consider their topological analogues. Iwasawa theory is the study of the asymptotic growth of class numbers in certain infinite abelian towers of number fields. This asymptotic growth is determined by Iwasawa invariants associated to the number field. Given a knot in a 3 sphere, a fundamental polynomial invariant is the Alexander polynomial; which is regarded as the analogue of the Iwasawa polynomial from number theory. We consider certain natural families of links embedded in the 3-sphere for which we prove results about the distribution of Iwasawa invariants. In greater detail, we consider distribution questions for the family of 2-bridge links. In time permits we shall also talk about the family of links associated to certain braids on a fixed number of strings.

• Xin Ren (Kansai University)

q-deformed rational numbers, q-deformed Farey sum and a triangulated category of A_2 quiver

Let q be a formal parameter. The left and right q-deformed rational numbers were introduced by Bapat, Becker and Licata via regular continued fractions, and the right q-deformed rational number is exactly q-deformed rational number considered by Morier-Genoud and Ovsienko [2], when q is a formal parameter. They gave a homological interpretation for left and right q-deformed rational numbers [1; Theorems 4.7 and 4.8].

In this talk, we give a formula for computing the q-deformed Farey sum of the left q-deformed rational numbers based on the negative continued fractions. We use this formula to give a combinatorial proof of the relationship between the left q-deformed rational number and the Jones polynomial of the corresponding rational knot which was proved by Bapat, Becker and Licata using a homological technique. We combine the homological interpretation of the left and right q-deformed rational numbers and the q-deformed Farey sum, and give a homological interpretation of the q-deformed Farey sum.

References

[1] Bapat, A., Becker, L., Licata, A. M.: q-deformed rational numbers and the 2-Calabi–Yau category of type A_2, Forum Math. Sigma 11 (2023), Paper No. e47, 41 pp.

[2] Morier-Genoud, S., Ovsienko, V.: q-deformed rationals and q-continued fractions, Forum Math. Sigma 8 (2020), Paper No. e13, 55 pp.

[3] Ren, X.: On q-deformed Farey sum and a homological interpretation of q-deformed real quadratic irrational numbers, arXiv:2210.06056, 2022.

• Mai Katada (Kyushu Universsity)

The first homology of the IA-automorphism groups of free groups with coefficients in spaces of Jacobi diagrams

The Kontsevich invariant for bottom tangles takes values in the spaces of Jacobi diagrams on oriented arcs. We have an action of the automorphism group of the free group on the space of Jacobi diagrams. We study the first homology of the IA-automorphism group, which is a normal subgroup of the automorphism group of the free group, with coefficients in the space of Jacobi diagrams of degree 2.

• Benjamin Collas (RIMS, Kyoto University)

Oda's prediction, a motivating conjecture

The action of the absolute Galois group of rational numbers on the (pro-ell) fundamental group of moduli spaces of curves of genus g with m marked points indicates general arithmetic properties for curves. One of those is given by Oda's prediction (OP; MPI Bonn 1993), which states the independence in g and m of the stable number field associated with these actions.

Oda's prediction has since acted as a motivating conjecture in arithmetic geometry – in terms of Lie algebra, divisorial arithmetic, or combinatorial an-

abelian geometry (resp. Matsumoto and Takao, Ihara-Nakamura, and Hoshi-Mochizuki), but also in low dimensional topology (Johnson homomorphism and Morita's obstruction; Morita, Nakamura, Hain, Takao et al.)

Our goal will be to provide a panorama of techniques and results that will also serve as a motivating introduction to Philip's talk on the application of stack arithmetic to Oda's prediction.

• Séverin Philip (RIMS, Kyoto University)

The maximal degeneration method for special loci

Collas and Maugeais classified the irreducible components of the cyclic special loci, a moduli space for curves admitting an action from a cyclic group. We will introduce an analogue of Oda's problem for this context and then explain the relations between the fields involved in this problem and the one from the classical Oda's problem. From this point we will see how an adaptation of the method of maximal degeneration from Ihara and Nakamura can be used to solve the problem.

• Yasuhiro Wakabayashi (Osaka University)

Topological quantum field theory for differential equations in characteristic p^N

We will discuss the enumerative geometry of linear differential equations (or more generally, opers) on algebraic curves in prime-power characteristic p^N. One of the central themes is to know how many such equations have a full set of solutions. In the case of $N = 1$, this counting problem has been largely resolved by effectively using connections with the Gromov-Witten theory of relative Grassmannians, the conformal field theory of affine Lie algebras, and combinatorics of rational polytopes, etc. Our recent work is an attempt to extend this theory to larger N.

• Nadav Gropper (University of Pennsylvania/University of Haifa)

TQFTs for pro-p Poincaré duality groups

In the talk, I will discuss the Turner-Turaev formalism for unoriented Topological Quantum Field Theory (TQFT). Building upon this formalism, I will introduce an analogous version for $(d+1)$-dimensional TQFT for pro-p Poincaré duality groups. In the case of $d = 1$, this enables us to study cobordisms and TQFTs for both the maximal pro-p quotient of absolute Galois groups of

p-adic fields and pro-p completions of fundamental groups of surfaces. This generalisation gives a framework for arithmetic TQFTs and strengthens the analogies within arithmetic topology, which relates p-adic fields to surfaces (oriented mod p^r). I will also provide an outline of the classification of such TQFTs for $d = 1$, in terms of Frobenius algebras with some extra structure. The talk is based on joint work with Oren Ben-Bassat.

• Yuki Ishida (Kyushu University)

The density of Borromean primes

In this talk, I will show that the density of Borromean primes is 1/128 among all of the triples of primes. *Borromean primes* are analogues of " Borromean rings " in topology. This analogy depends on the relationship between number theory and 3-dimensional topology in " arithmetic topology", originated by B. Mazur, M. Morishita, M. Kaparanov and A. Reznikov. The idea of *density* of primes lies in " arithmetic statistics", which has been developed from Prime Number Theorem. Our study is in the intersection of these areas. I will overview these two areas and briefly explain my main theorem and numerical results.

• Yuqui Deng (Kyushu University)

Arithmetic Dijkgraaf–Witten invariants for real quadratic fields, quadratic residue graphs, and density formulas

We compute Hirano's formula for the mod 2 arithmetic Dijkgraaf–Witten invariant (partition function) Z_k for the ring of integers of the quadratic field $k = \mathbb{Q}(\sqrt{p_1}, \ldots, \sqrt{p_r})$, where p_i's are distinct prime numbers with $p_i \equiv 1 \pmod 4$, and give a simple formula for Z_k in terms of the graph obtained from quadratic residues among $p_1, \ldots, p_r$. Our result answers the question posed by Ken Ono. We also give a density formula for mod 2 arithmetic Dijkgraaf–Witten invariants.

• Toshiki Matsusaka (Kyushu University)

Introduction to quantum modular forms and discontinuity property of Habiro series

• Hisatoshi Kodani (IMI, Kyushu University)

Quantum master equation and Hodge correlators

We give a generalization of Goncharov's Hodge correlator twistor connection. Our generalized version is a connection 1-form with values in a DG Lie algebra of uni-trivalent graphs which may have loops and satisfies some Maurer–Cartan equation. This connection and the Maurer–Cartan equation can be viewed as an arithmetic analogue of effective action and quantum master equation respectively in non-acyclic Chern–Simons perturbation theory around trivial local system. This is joint work with Yuji Terashima.

• Soichiro Uemura (Kavli IPMU, The University of Tokyo)

A proof of the Teichmüller TQFT volume conjecture for the knot 7_3

We will introduce the generalized topological quantum field theory (TQFT) constructed by Andersen and Kashaev. This TQFT is based on the quantum Teichmüller theory and is called the Teichmüller TQFT. The Teichmüller TQFT is expected to be a mathematical model for the $SL(2,\mathbb{C})$ Chern-Simons theory. In the Teichmüller TQFT, a partition function is defined for a triangulated 3-manifold with an angle structure. As for the partition function, Andersen and Kashaev proposed a conjecture analogous to the volume conjecture for the pair of any closed 3-manifold and any embedded hyperbolic knot. Aribi, Guéritaud and Piguet-Nakazawa proved this conjecture in a reformulated form for the pair of the three-dimensional sphere S^3 and any twist knot. We proved a theorem similar to the volume conjecture for the pair of S^3 and a hyperbolic knot 7_3, which does not belong to the family of twist knots (arXiv: 2307.12848). We will give an outline of the proof.

• Jun Ueki (Ochanomizu University)

Arithmetic topology and the profinite rigidity of the multivariable Alexander polynomials of links

Let p be a prime number and d a positive integer. We establish several versions of the Iwasawa-type formula for $\mathbb{Z}_p^d$-covers of links in rational homology 3-spheres. We also discuss the profinite rigidity of the multivariable Alexander polynomials by using the same tools, say, the fundamental two exact sequences. (based on joint works with Sohei Tateno and Biao Ma.)

• Hiraku Nozawa (Ritsumeikan University)

Harmonic measures and rigidity for the Fuchsian actions of surface groups on the circle

A measure on a foliated manifold equipped with a leafwise Riemannian metric is harmonic if it is invariant under the leafwise heat flow. In an unfinished article, Thurston proposed a construction of an S^1 connection on an S^1-bundle over a hyperbolic surface by using a harmonic measure on a foliation transverse to the S^1-fibers. We will explain that, even if the surface has cusps, the Gauss-Bonnet formula holds for this S^1 connection and the bounded Euler number due to Burger-Iozzi-Wienhard. As an application, we will give an alternative proof of Matsumoto rigidity theorem of Burger-Iozzi-Wienhard for the Fuchsian actions of hyperbolic surfaces with cusps. This talk is based on a joint work with Masanori Adachi (Shizuoka Univ.) and Yoshifumi Matsuda (Aoyama Gakushuin Univ.).