Low dimensional topology and number theory XIV

IMI Auditorium (413 Rm), 4F West 1st Bd, Kyushu University (Ito Campus)
27th March, 2023 $\sim$ 30th March, 2023

Program

March 27th (Mon)
9:30 – 10:30
Jun Ueki (Ochanomizu University)
Non-acyclic SL_2-representations, surgeries, and L-functions of the twisted Whitehead links

10:50 – 11:50
Sohei Tateno (Nagoya University)
The Iwasawa invariants of $\mathbb{Z}_p^d$-covers of links

13:40 – 14:40
Takefumi Nosaka (Tokyo Institute of Technology)
Reciprocity of the Chern-Simons invariants of 3-manifolds

15:00 – 16:00
Stavros Garoufalidis (Southern University of Science and Technology)
Three realizations of holomorphic quantum modular forms

March 28th (Tues)
9:30 – 10:30
Naganori Yamaguchi (RIMS, Kyoto University)
On the development of anabelian geometry using the maximal geometrically m-step solvable quotient of arithmetic fundamental groups

10:50 – 11:50
Shun Ishii (RIMS, Kyoto University)
On pro-p outer Galois representations associated to once-punctured CM elliptic curves

13:40 – 14:40
Takeo Uramoto (IMI, Kyushu University)

On the modularity theorem for algebraic Witt vectors

15:00 – 16:00
James Borger (Australian National University)
Report on scheme theory over semirings

March 29th (Wednes)
9:30 – 10:30
Yosuke Morita (Kyoto University)
A new framework for Conley index theory

10:50 – 11:50
Jonathan Beardsley (University of Nevada)
Toward Higher Algebra Over $\mathbb{F}_1$

13:40 – 14:40
Takuya Takeishi (Kyoto Institute of Technology)
Rigidity theorems of C*-algebras arising from number theory

15:00 – 16:00
Takeshi Shinohara and Nao Komiyama (Nagoya University)
Shuffle product of desingularized multiple zeta functions at integer points

March 30th (Thurs)
10:00 – 11:00
Atsushi Katsuda (Kyushu University)
Closed geodesics, Heat kernels and Hofstadter butterfly

13:00 – 14:00
Jesus Ántonio Alvarz López (Universidade de Santiago de Compostela)
A trace formula for foliated flows

14:20 – 15:20
Dohyeong Kim (Seoul National University)
Iwasawa's invariant and its Diophantine application

Abstract

• Jesús A. Álvarez López (University of Santiago de Compostela)

A trace formula for foliated flows

Let $(M, \mathcal{F}, \phi^t)$ be a smooth compact manifold equipped with codimension one foliation and a foliated flow (the flow maps leaves to leaves). Assume ϕ^t has simple closed orbits and transversely simple preserved leaves. Then there are finitely many leaves preserved by the flow, whose union is a compact submanifold M^0, and a precise description of the transverse structure of $\mathcal{F}$ can be given. The distributional leafwise differential forms conormal to M^0 form a complex with the de Rham derivative of the leaves, giving rise to the conormal leafwise reduced cohomology $\bar{H}I(\mathcal{F})$. We define a Leftchetz distribution $L_{\mathrm{dis}}(\phi^t)$ on $\mathbb{R}$ of the induced action ϕ^{t*} on $\bar{H}I(\mathcal{F})$. Then we prove a distributional Lefschetz trace formula describing $L_{\mathrm{dis}}(\phi^t)$ in terms of infinitesimal data of the closed orbits and preserved leaves. This kind of distributional trace formula was conjectured by Christopher Deninger, motivated by possible arithmetic interpretations. In the case where $M^0 = \emptyset$ (when $\mathcal{F}$ is Riemannian), this formula was proved using smooth leafwise differential forms. This is joint work with Yuri Kordyukov and Eric Leichtnam.

• Jonathan Beardsley (University of Nevada)

Toward Higher Algebra Over $\mathbb{F}_1$

Abstract: In a series of papers, Alain Connes and Caterina Consani have introduced a framework for doing "algebra in characteristic one." Their approach has strong connections to Segal's approach to stable homotopy theory via so-called Γ-spaces. I will describe some first steps in further developing this analogy. In particular I will show that within their framework there is a natural notion of the "classifying space" of an $\mathbb{F}_1$-module. If A is the $\mathbb{F}_1$-module associated to an abelian group, commutative monoid, or abelian hypergroup, with classifying space BA, then taking the loop space of BA (for a slightly non-standard notion of loop space described by Connes and Consani) recovers A. This suggests that higher deloopings and possibly Eilenberg-MacLane spectra of $\mathbb{F}_1$-modules may be possible, though they would require working with a suitable model for (∞,∞)-categories. This is joint work with Joe Moeller.

• James Borger (Australian National University)

Report on scheme theory over semirings

Usually in algebraic geometry, one works with schemes defined over base fields. But for arithmetic applications, where transferring information between positive characteristic and characteristic zero is necessary, it's convenient to work with schemes defined over base rings, such as the integers or the p-adic integers. Indeed, since this formalism became available in the mid-20th century, it has become standard.

It is however possible to go even deeper and set up scheme theory over any base semiring. This includes objects like projective space over the non-negative reals, the Boolean numbers, or the natural numbers, which is the deepest subring of all. Doing so allows us to preserve positivity information in the underlying theory, much as the passage from base fields to base rings allowed for the preservation of integrality information.

Scheme theory over semirings is however in its infancy. In this talk, I'll report on some areas of scheme theory over rings which have been recently extended to semirings, such as the etale fundamental group (joint with Robert Culling) and the Picard stack (joint with Jaiung Jun). One notable point that came out of the latter work is that the various familiar definitions of vector bundle in scheme theory over rings do not remain equivalent over semirings but that all the familiar definitions of line bundle do remain equivalent.

• Stavros Garoufalidis (Southern University of Science and Technology)

Three realizations of holomorphic quantum modular forms

We will discuss three realizations of holomorphic quantum modular forms that appeared in quantum topology, as matrix-valued functions defined at roots of unity, as factorially divergent formal power series and as holomorphic functions on a complex cut plane. This is joint work with D. Zagier.

• Shun Ishii (RIMS, Kyoto University)

On pro-p outer Galois representations associated to once-punctured CM elliptic curves

Let p be an odd prime. Sharifi proved that, under the Deligne-Ihara conjecture (now a theorem of Brown), the kernel of the pro-p outer Galois representation associated to the thrice-punctured projective line corresponds to the maximal pro-p extension of $\mathbb{Q}(\mu_p)$ unramified outside p if p is regular. In this talk, we discuss an analogue of his result for imaginary quadratic fields by considering once-punctured CM elliptic curves instead of the thrice-punctured projective line.

• Atsushi Katsuda (Kyushu University)

Closed geodesics, Heat kernels and Hofstadter butterfly

The Bloch-Floquet theory are popular tools for the investigation of materials with periodic structures. For example, we can show that the spectrum of periodic Schrödinger operators have band structures. In the context of this talk, this theory was applied to the following problems in the case of abelian extensions:

(1) A geometric analogue of the Chebotarev density theorem for prime closed geodesics in a compact Riemannian manifold with negative curvature

(2) A long time asymptotic expansion of the heat kernels of covering manifolds of compact Riemannian manifolds.

Here, we shall extend the above results by applying our version of the Bloch-Floquet theory for the Heisenberg group, which is the simplest, but non-trivial non-abelian infinite group. Our method is based on a combination of the representation theory for discrete Heisenberg groups especially due to Pytlik and that of the Heisenberg Lie group.

Moreover, there is a relation to the our methods for the Heisenberg group and the analysis of the discrete magnetic Laplacian or the Harper operator on the square lattice $\mathbb{Z}^2$. Spectrum of the latter operators are expressed by the celebrated Hofstadter butterfly as the following figure:

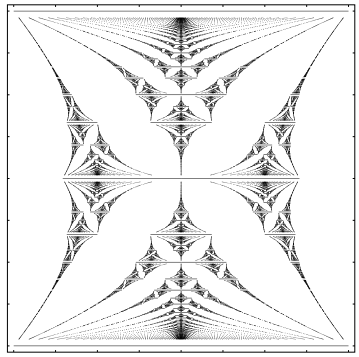

Figure 1: the Hofstadter's butterfly (created by Hisashi Naito).

Especially, our arguments give an another mathematical justification of semi-classical asymptotic expansion formula for spectrum of the Harper operator due to Wilkinson, which is originally done by Helffer-Sjöstrand.

$$E_n = -4 + (2n+1)\theta + O(\theta^2) \qquad n = 0, 1, 2, \ldots l$$

If time permits, I shall explain generalizations of our method to more general nilpotent groups and further applications.

• Dohyeong Kim (Seoul National University)

Iwasawa's invariant and its Diophantine application

We review the cyclotomic Iwasawa theory and apply the vanishing of the mu-invariant to some Diophantine problems. I will compare the vanishing of mu-invariants to the monicity of twisted Alexander polynomials.

• Nao Komiyama, Takeshi Shinohara (Nagoya University)

Shuffle product of desingularized multiple zeta functions at integer points

We show that the "shuffle-type" formula holds for special values of desingularized multiple zeta functions at any integer points. This is proven by giving an iterated/differential expression for the special values at integer points of the desingularized multiple zeta functions. We explain how the so-called renormalization method is applied in our proof.

• Yosuke Morita (kyoto University)

A new framework for Conley index theory

The Conley index is, informally speaking, a 'space' that describes the local dynamics around an isolated invariant subset of a topological dynamical system. It can be seen as a spatial refinement of the Morse index and is used, for instance, in Manolescu's construction of the Seiberg-Witten-Floer homotopy type. In this talk, I will explain a new framework for Conley index theory, which I think is simpler and more flexible than the traditional formulation. One important point is that the Conley index should be defined as a based equivariant ind-(compact Hausdorff space) (or slightly more generally, a based equivariant condensed set/anima), not as a mere homotopy type of topological spaces.

• Takefumi Nosaka (Tokyo Institute of Technology)

Reciprocity of the Chern-Simons invariants of 3-manifolds

Given an oriented closed 3-manifold M and a representation $\pi_1(M) \longrightarrow SL_2(\mathbb{C})$, we can define the Chern-Simons invariant and adjoint Reidemeister torsion. Recently, several physicists and topologists pose and study reciprocity conjectures of the torsions. Analogously, I posed reciprocity conjectures of the Chern-Simons invariants of 3-manifolds, and discussed some supporting evidence on the conjectures. Especially, I showed that the conjectures hold if Galois descent of a certain K_3-group is satisfied. In this talk, I will explain the background and the results in detail.

• Takuya Takeishi (Kyoto Institute of Technology)

Rigidity theorems of C*-algebras arising from number theory

C*-algebras can be constructed from various mathematical objects, and it is commonly interested to detect what kind of information is preserved by taking C*-algebras. Usually, a lot of information may vanish, but it turned out that several classes of C*-algebras constructed from number fields remember the original number fields completely. As a consequence, we can construct several classes of countable (noncommutative) groups which are complete invariants of number fields. In this talk, we explain the speaker's recent contribution to the study of this phenomenon, together with several related results.

• Sohei Tateno (Nagoya University)

The Iwasawa invariants of $\mathbb{Z}_p^d$-covers of links

In this talk, we will define the Iwasawa invariants of links and give two asymptotic formulae for the first homology groups of $\mathbb{Z}_p^d$-covers of links in rational homology 3-spheres, which are generalizations of the Iwasawa type formulae proven by Hillman-Matei-Morishita and Kadokami-Mizusawa. We will also provide examples of these formulae. Moreover, when $d = 2$, considering the twisted Whitehead links, we will explain that Iwasawa μ-invariants can be arbitrary non-negative integers. This is a joint work with Jun Ueki.

• Jun Ueki (Ochanomizu University)

Non-acyclic SL_2-representations, surgeries, and L-functions of the twisted Whitehead links

We study the zeros of the SL_2-acyclic torsion functions τ of links. We first study that of the Whitehead link W_1 and deduce results for twist knots simultaneously. Next, we extend the study to every twisted Whitehead link W_k with $k \in \mathbb{Z}$ and prove that the zeros of τ have the multiplicity two on the geometric component of the character variety. Finally, we paraphrase the notion of multiplicity of common zeros and investigate the L-functions of the universal deformations of residual representations of W_k. (Based on a joint work with Leo Benard, Ryoto Tange, and Anh Tran.)

• Takeo Uramoto (IMI, Kyushu University)

On the modularity theorem for algebraic Witt vectors

In my previous work, I proved the (weak/strong) "modularity theorem for algebraic Witt vectors" over imaginary quadratic fields K, the strong one of which claims that they are precisely those generated over K by modular vectors whose coefficients are special values of deformation family of Fricke modular functions. Arithmetically this construction implies, say, congruences between special values of modular functions living at different levels (rather, not necessarily galois-conjugate ones), and provides natural re-interpretations of classical objects (say, Fricke functions, Witt vectors) in class field theory and the theory of complex multiplication, albeit it yet seems to be a special facet of some more general theme of deformation theory. In view of this, I would like to devote this talk to some clarification/discussion on more generic geometric structures behind this specific phenomenon, extending it to the case of CM fields in particular.

• Naganori Yamaguchi (RIMS, Kyoto University)

On the development of anabelian geometry using the maximal geometrically m-step solvable quotient of arithmetic fundamental groups

In anabelian geometry, we have the following conjecture called the Grothendieck conjecture: The geometric information of hyperbolic curves is reconstructed group-theoretically from their arithmetic fundamental groups. This conjecture was proved by Hiroaki Nakamura, Akio Tamagawa, and Shinichi

Mochizuki. However, many unresolved problems still remain around this conjecture. One of these problems is the main topic of this talk which is called the m-step solvable Grotheindieck conjecture, specifically: The geometric information of hyperbolic curves is reconstructed group-theoretically from the maximal geometrically m-step solvable quotient of their arithmetic fundamental groups. In this talk, we will present about this conjecture and a part of its proof by the speaker.