Cut elimination and semi-completeness

A standard way of giving a syntactic proof of cut elimination comprises combinatorial analysis of proof structures in a given sequent system and constructive procedures for eliminating each application of cut rule. Meanwhile, a semantical proof of cut elimination is obtained usually by showing the completeness of a sequent system without cut rule. Thus, if it is a model-theoretic one, i.e. a semantical proof using Kripke frames, a counter-model will be given for each sequent which is not provable in such a cut-free system. This is done by extending a *partial valuation* to a total one (Schütte's idea). This has been successfully carried out for sequent systems for modal logics.On the other hand, algebraic proofs can be applied also to sequent systems for substructural logics. In this case, it is shown that every *Gentzen structure* (with a quasi-order, which corresponds to a cut-free system) can be *quasi-embedded* into an algebra for the full system so that "non-provability" is preserved. It got noticed that it has a close connection with MacNeille completions.

The aim of my talk is to establish a link between these two semantical approaches, by introducing the notion of *semi-completeness* of sequent systems, which came out of S. Maehara's paper in 1991. As a case study, we discuss a sequent system for modal (predicate) logic **S4**.