Young Mathematicians Workshop on Several Complex Variables 2024

Room E408, Faculty of Science Building E, Sugimoto Campus,
Osaka Metropolitan University*

August 5–8, 2024

Monday, August 5

9:30–10:10 **Seungjae Lee** (Kyungpook National University)
An L^2-Dolbeault type Hodge theory on holomorphic ball quotient with finite volume

10:30–11:10 **Yoshiaki Suzuki** (Niigata University)
The eigenvalues and eigenfunctions of the Folland–Stein operator on some Heisenberg Bieberbach manifolds

11:30–12:10 **Song-Yan Xie** (Academy of Mathematics and Systems Science)
Hole probabilities of random zeros on compact Riemann surfaces

Lunch

13:40–14:20 **Takayuki Watanabe** (Chubu University)
New variants of Newton's method from the complex dynamical viewpoint

14:40–15:20 **Zhangli Lin** (Xiamen University)
The subgroups generated by simple holomorphic automorphisms of $(\mathbb{C}^*)^n$

15:40–16:20 **Feng Rong** (Shanghai Jiao Tong University)
A brief survey on the Burns–Krantz type rigidity

16:40–17:20 **Jihun Yum** (Gyeongsang National University)
Bergman local isometries are biholomorphisms

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Tuesday, August 6

9:30–10:10 **Shuho Kanda** (The University of Tokyo)
The hard Lefschetz condition for locally conformally symplectic manifolds

10:30–11:10 **Kyeong-Dong Park** (Gyeongsang National University)
Greatest Ricci lower bounds of odd symplectic Grassmannians

11:30–12:10 **Minseong Kwon** (KAIST / IBS-CCG)
Deformation of twistor conics in homogeneous Fano contact manifolds

Lunch

Free Discussion

Wednesday, August 7

9:30–10:10 **Hoseob Seo** (IBS-CCG)
On equisingular approximations of toric plurisubharmonic functions and currents

10:30–11:10 **Kuang-Ru Wu** (National Tsing Hua University)
Positivity for the tensor product of vector bundles

11:30–12:10 **Xun Sun** (Peking University)
The solution of the Suita conjecture and its generalization

Lunch

13:40–14:20 **Wang Xu** (Sun Yat-sen University)
Optimal L^2 extension of openness type and related topics

14:40–15:20 **Shota Kikuchi** (National Institute of Technology, Suzuka College)
On advances in the Berndtsson–Lempert type L^2-extension theorem by using pluricomplex Green functions

15:40–16:20 **Xieping Wang** (University of Science and Technology of China)
L^2 extension theorem and removable singularities of plurisubharmonic functions

Banquet

Thursday, August 8

9:30–10:10 **Takahiro Aoi** (National Institute of Technology, Wakayama College)
Skoda–Zeriahi type integrability for some measure with L^1-density and its application to relative entropy

10:30–11:10 **Bowoo Kang** (KAIST)
On the bounded subsolution problem for the complex Monge–Ampère flows

11:30–12:10 **Rei Murakami** (Tohoku University)
Weak limits of the J-flow and the cotangent flow

Abstracts

Skoda–Zeriahi type integrability for some measure with L^1-density and its application to relative entropy

Takahiro Aoi (National Institute of Technology, Wakayama College)

I will talk about some integrability result of plurisubharmonic functions for some measure with L^1-density. In order to prove this, Skoda–Zeriahi's integrability theorem and the Ohsawa–Takegoshi L^2-extension theorem play a very important role. As an application, we show some compactness for relative entropy. This work is motivated by the existence problem of constant scalar curvature Kähler metrics of Poincaré type.

The hard Lefschetz condition for locally conformally symplectic manifolds

Shuho Kanda (The University of Tokyo)

The hard Lefschetz condition (HLC) is a useful classical tool for determining whether a symplectic manifold admits a Kähler structure. We generalize the HLC for locally conformally symplectic manifolds, which are generalizations of symplectic manifolds. For applications, it is expected that every locally conformally Kähler manifold satisfies the HLC; however, this remains a conjecture. In this talk, we explain the significance of the proposed condition, the difficulty of the conjecture, and some partial results.

On the bounded subsolution problem for the complex Monge–Ampère flows

Bowoo Kang (KAIST)

The complex Monge–Ampère flow is a parabolic PDE modeled on the Kähler–Ricci flow. In 2021, Guedj–Lu–Zeriahi constructed a parabolic pluripotential theory to study degenerate versions of complex Monge–Ampère flows. It is a natural question whether results of the elliptic pluripotential theory can be extended to the parabolic pluripotential theory. In this talk, I shall present my recent results on the bounded subsolution problem for the complex Monge–Ampère flows. This is motivated by the result in elliptic theory by Kołodziej (1995).

On advances in the Berndtsson–Lempert type L^2-extension theorem by using pluricomplex Green functions

Shota Kikuchi (National Institute of Technology, Suzuka College)

Berndtsson and Lempert gave the optimal L^2-extension theorem which is based on variational theory of pseudoconvex domains in $\mathbb{C}^n$. This result is called the Berndtsson–Lempert type L^2-extension theorem.

The Berndtsson–Lempert type L^2-extension theorem is obtained through the monotonicity property of minimal L^2-extensions. In particular, it is known that this monotonicity property of minimal L^2-extensions is proved by the Berndtsson's convexity theorem. In addition, plurisubharmonic functions with poles along subvarieties play an important role.

In this talk, I explain recent advances in the Berndtsson–Lempert type L^2-extension theorem by using pluricomplex Green functions.

Deformation of twistor conics in homogeneous Fano contact manifolds

Minseong Kwon (KAIST / IBS-CCG)

Twistor spaces are complex contact manifolds arising from a certain class of real Riemannian manifolds, so-called quaternionic Kähler manifolds. When the given quaternionic Kähler manifold has a positive scalar curvature and is compact, then its associated twistor space becomes a Fano manifold, hence an algebro-geometric method can be applied. In this talk, first I will review the construction of the twistor space, together with its foliation by Riemann spheres called twistor conics. Next, after recalling LeBrun–Salamon conjecture which predicts that every Fano contact manifold is homogeneous, I will give a description of all possible deformation of twistor conics when the given twistor space is Fano and homogeneous.

An L^2-Dolbeault type Hodge theory on holomorphic ball quotient with finite volume

Seungjae Lee (Kyungpook National University)

In this talk, I will introduce an L^2-Dolbeault type version of the Hodge theory for the symmetric power of holomorphic cotangent bundle $S^m T^*_\Sigma$ of a complex unit ball quotient with finite volume Σ. It is well-known that when a Kähler manifold has a d-bounded Kähler potential, for a Nakano semi-negative holomorphic vector bundle E, an L^2-Dolbeault type Hodge decomposition on E-valued $(0,q)$ forms holds. However, the symmetric power $S^m T^*_\Sigma$ is not Nakano semi-negative when

the degree m is small. Therefore, it is not possible to directly apply the result. To overcome the difficulty, Seo and I developed a variation of the approach that was initially used to establish vanishing theorems for logarithmic sheaves of simple normal crossing divisors on a compact Kähler manifold. This presentation is based on a joint work with Aeryeong Seo of Kyungpook National University.

The subgroups generated by simple holomorphic automorphisms of $(\mathbb{C}^*)^n$

Zhangli Lin (Xiamen University)

Let $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$ and $\mathrm{Aut}_\omega((\mathbb{C}^*)^n)$ be the group consisting of holomorphic automorphisms of $(\mathbb{C}^*)^n$ that preserves the invariant volume form $\omega = (z_1 \cdots z_n)^{-1} dz_1 \wedge \cdots \wedge dz_n$. In this talk we consider several subgroups of $\mathrm{Aut}_\omega((\mathbb{C}^*)^n)$ generated by different types of simple automorphisms. The first is the subgroup consisting of those automorphisms that can be extended to automorphisms of $\mathbb{C}^n$. The second is a smaller subgroup — those automorphisms that fix every point of the coordinate hyperplane. The last is $\mathrm{Aut}_\omega((\mathbb{C}^*)^n)$ itself. Our main results are three Andersen–Lempert type theorems, which answer an open question posed by Rosay and Rudin in 1988. This is a joint work with Xiangyu Zhou.

Weak limits of the J-flow and the cotangent flow

Rei Murakami (Tohoku University)

The J-equation and the Leung–Yau–Zaslow equation are partial differential equations related to canonical metrics in Kähler geometry. The solvability of these equations is known to be equivalent to a certain type of numerical positivity. In this talk, we consider the case where "the numerical semipositivity" satisfies on a compact Kähler surface. We prove that in this case, the corresponding flows converge to weak solutions of them.

Greatest Ricci lower bounds of odd symplectic Grassmannians

Kyeong-Dong Park (Gyeongsang National University)

The odd symplectic Grassmannian $\mathrm{SGr}(k, 2n+1)$ is defined as a variety parametrizing k-dimensional isotropic subspaces in a $(2n+1)$-dimensional complex vector space equipped with a skew-symmetric bilinear form of maximal rank. We know that these are smooth Fano varieties and the automorphism groups of nonhomogeneous odd symplectic Grassmannians are non-reductive, which implies that they

admit no Kähler–Einstein metrics. As a numerical measure of the extent to which a Fano manifold is close to be Kähler–Einstein, we explicitly compute the greatest Ricci lower bounds of odd symplectic Grassmannians using the barycenter of each moment polytope with respect to the Duistermaat–Heckman measure. In particular, the greatest Ricci lower bound of the odd symplectic Grassmannian $\mathrm{SGr}(n, 2n+1)$ can be arbitrarily close to zero as n grows.

A brief survey on the Burns–Krantz type rigidity

Feng Rong (Shanghai Jiao Tong University)

We will first recall the original Burns-Krantz rigidity and Huang's many contributions. Then, we will go over some of our recent results, including the proof of Huang's conjecture, the boundary rigidity on fibered domains, the n-point boundary rigidity, etc.

On equisingular approximations of toric plurisubharmonic functions and currents

Hoseob Seo (IBS-CCG)

When a plurisubharmonic function on a complete Reinhardt domain is invariant under the torus action, convex analysis plays a crucial role in analyzing its properties. For instance, the multiplier ideal sheaf, the log canonical threshold of it at the origin and the existence of equisingular approximations with analytic singularities can be described in terms of its Legendre transformation. In this talk, we will review the correspondence between toric positive closed currents and real positive closed currents and then show the existence of decreasing equisingular approximations of a toric positive closed $(1,1)$-current converging in the Hartogs sense.

The solution of the Suita conjecture and its generalization

Xun Sun (Peking University)

The Suita conjecture is about a relation between the magnitudes of the Bergman kernel and the logarithmic capacity of an open Riemann surface with respect to a fix point, which was posed by Suita in 1972. The inequality part of the Suita conjecture was solved by Błocki in the situation of plane region. The inequality part for general situation and the equality part was solved by Guan–Zhou. Recently, Guan–Mi–Yuan jointly gave several generalizations of the Suita conjecture,

Guan, Yuan and I gave several characterizations of the equality part of the generalized Suita conjecture. I will survey the solution and generalization of the Suita conjecture, and introduce our recent work.

The eigenvalues and eigenfunctions of the Folland–Stein operator on some Heisenberg Bieberbach manifolds

Yoshiaki Suzuki (Niigata University)

Heisenberg Bieberbach manifolds are compact quotients of the Heisenberg group by a discrete torsion-free subgroup of the semidirect product of the Heisenberg group and the unitary group. Any Heisenberg Bieberbach manifold has a natural CR structure induced by the standard CR structure on the Heisenberg group. In this talk, we study the eigenvalues and eigenfunctions of the Folland–Stein operator (gives the Kohn Laplacian essentially) on 3-dimensional Heisenberg Bieberbach manifolds. In 2004, Folland determined the eigenvalues and eigenfunctions of the Folland–Stein operator on the compact quotients of the Heisenberg group by a lattice subgroup. Using Folland's method, we calculate the eigenvalues and the dimensions of the eigenspaces of the Folland–Stein operator on some examples of Heisenberg Bieberbach manifolds.

L^2 extension theorem and removable singularities of plurisubharmonic functions

Xieping Wang (University of Science and Technology of China)

I will present a removable singularity theorem for psh functions across a compact complete pluripolar set in Stein manifolds. To put this result in historical context, I will also review several related classical results on psh functions and closed positive currents. If time permits, I will also outline the proof of our theorem, which involves a version of the Ohsawa–Takegoshi L^2 extension theorem on complete Kähler manifolds and some basic geometric measure theory.

New variants of Newton's method from the complex dynamical viewpoint

Takayuki Watanabe (Chubu University)

Newton's method is the most well-known algorithm which produces an approximation to the root of a given function if an initial point is chosen near a root. The initial values for which Newton's method converges or fails are known to form

beautiful fractals, which led to the development of the theory of complex dynamical systems. In this talk, we introduce new variants of Newton's method and compare their structures with Voronoi diagrams and the so-called Newton's flow. This talk is partly based on joint work with John Erik Fornæss, Mi Hu, and Tuyen Trung Truong.

Positivity for the tensor product of vector bundles

Kuang-Ru Wu (National Tsing Hua University)

It is known that, for holomorphic vector bundles, ampleness and Griffiths positivity are preserved under tensor product. In this talk, we will show that Kobayashi positivity is also preserved under tensor product. This result provides further evidence on the conjectural equivalences between ampleness, Griffiths positivity, and Kobayashi positivity.

Hole probabilities of random zeros on compact Riemann surfaces

Song-Yan Xie (Academy of Mathematics and Systems Science)

We establish a convergence speed estimate for hole probabilities of zeros of random holomorphic sections on compact Riemann surfaces. The proof is based on a density formula of Zelditch, the Abel–Jacobi theory, Fekete points theory, and a new perturbation method. This is joint work (arXiv:2406.19114) with Hao Wu (NUS).

Optimal L^2 extension of openness type and related topics

Wang Xu (Sun Yat-sen University)

L^2 extension theorems and optimal L^2 extension theorems are important and powerful tools in several complex variables and complex geometry. There is a closely related problem called the optimal L^2 extension problem of openness type: given a holomorphic section f defined on a neighbourhood U of a subvariety S, find a holomorphic extension of $f|_S$ to the ambient manifold, whose L^2 norm is optimally controlled by the L^2 norm of f on U. In this talk, I will present a solution on weakly pseudoconvex Kähler manifolds, which generalizes a couple of known results. I will also discuss some connections and applications to related topics, such as sharper L^2 extensions and generalized Suita conjectures. This talk is mainly based on joint work with Prof. Xiangyu Zhou.

Bergman local isometries are biholomorphisms

Jihun Yum (Gyeongsang National University)

Let Ω_1 and Ω_2 denote bounded domains in $\mathbb{C}^n$ equipped with the Bergman metrics g_{B1} and g_{B2}, respectively. If a biholomorphism $f\colon \Omega_1 \to \Omega_2$ exists, then according to the transformation formula for the Bergman kernels, it is well-established that f induces an isometry with respect to the Bergman metric, i.e., $f^* g_{B2} = g_{B1}$. In this talk, we explore the converse implication. The main theorem is the following.

Let Ω_1 and Ω_2 be bounded domains in $\mathbb{C}^n$. For a proper holomorphic map $f\colon \Omega_1 \to \Omega_2$, if $f^* g_{B2} = \lambda g_{B1}$ holds on an open subset $U \subset \Omega_1$ for some constant $\lambda > 0$, then f is a biholomorphism.

The proof of the main theorem relies on a novel method grounded in Information Geometry theories: the Factorization Theorem for sufficient statistics and the result ([1]) established by G. Cho and myself.

[1] G. Cho and J. Yum, *Statistical Bergman geometry*, preprint arXiv:2305.10207 (2023).