# Japan - Taiwan Joint Workshop on Multiple Zeta Values 2020

## Abstracts

**February 8. (Saturday)**

Jing Yu    (National Taiwan University, Taiwan)

**Title:** Motivic Galois Groups and MultiZeta Values in Characteristic $p$

**Abstract:** We report our recent studies of MZV in positive characteristic. Let $\Gamma_{\mathbf{s}}$ be the motivic Galois group of the $t$-motive $M_{\mathbf{s}}$ associated to the depth $r$ MZV $\zeta_A(\mathbf{s})$, following Anderson-Thakur. Assuming a conjecture of Lara Rodriguez-Thakur on "slicing", we show that $\zeta_A(\mathbf{s})$ is Eulerian if and only if $\Gamma_{\mathbf{s}} \cong \mathbf{G}_m$. Also if the depth $r$ MZV $\zeta_A(\mathbf{s})$ is "zetalike ", then $\Gamma_{\mathbf{s}}$ is an extension of $\mathbf{G}_m$ by a vector group of dimension $\leq r$. The dimension of this vector group can be explicitly given in terms of the tuple $\mathbf{s}$.

Yen-Tsung Chen    (National Tsing Hua University, Taiwan)

**Title:** On the linear independence of $\theta$-adic multiple zeta values over function fields

**Abstract:** In this talk, we will talk about Chang-Mishiba $v$-adic multizeta values over function fields. We prove an analogue of the Goncharov's direct sum conjecture when $v$ is of degree one. The same result is also valid for $v$-adic Carlitz multiple polylogarithms at algebraic point when $v$ is of degree one.

Ryotaro Harada    (Nagoya University, Japan)

**Title:** A positive characteristic analogue of alternating multizeta values

**Abstract:** In 2004, Thakur introduced and studied a positive characteristic analogue of multizeta values. It is known that they have properties including non-vanishing property, sum-shuffle relation, period interpretation and linear independence by the work of Anderson, Chang and Thakur. In this talk, we will introduce and study alternating multizeta values in positive characteristic which give a certain generalization of Thakur's multizeta values. We will describe these objects have an alternating version of the above four properties. These results enable us to settle Goncharov's conjecture for alternating multizeta values in positive characteristic.

**February 9. (Sunday)**

Masataka Ono    (Kyushu University, Japan)

**Title:** On truncated versions of $t$-adic symmetric multiple zeta values

**Abstract:** The $t$-adic symmetric multiple zeta values ($t$-adic SMZVs), which was introduced by Hirose, Jarossay, and Rosen independently, is a symmetric counterpart of the $p$-adic finite multiple zeta values. In this talk, we give 2 types of truncated version of $t$-adic SMZVs and present their properties. This talk is based on joint works with Shin-ichiro Seki and Shuji Yamamoto.

Minoru Hirose    (Kyushu University, Japan)

**Title:** Motivic Galois group over $\mathbb{Z}[1/2]$ and linear relations among motivic alternating multiple zeta values

**Abstract:** Motivic alternating multiple zeta values are signed analogues of motivic multiple zeta values. In this talk, we introduce alternating analogues of the confluence relations, and show that they give all linear relations among motivic alternating multiple zeta values. Furthermore we explain that this result gives a complete answer to a $\mathbb{Z}[1/2]$ analogue of a well-known open conjecture that the motivic Galois group of mixed Tate motives over $\mathbb{Z}$ coincides with Grothendieck-Teichmuller group. This is a joint work with Nobuo Sato at Kyushu University.

Yen-Liang Kuan    (National Center for Theoretical Sciences, Taiwan)

**Title:** The Mordell-Weil theorem for $t$-modules

**Abstract:** For each positive characteristic multiple zeta value (defined by Thakur) $\zeta_A(\mathfrak{s})$, Chang-Papanikolas-Yu constructed the $t$-module $E_\mathfrak{s}$ defined over $A$ and integral points $\mathbf{v}_\mathfrak{s}$, $\mathbf{u}_\mathfrak{s} \in E_\mathfrak{s}(A)$. They proved that $\zeta_A(\mathfrak{s})$ is Eulerian (resp. zeta-like) if and only if $\mathbf{v}_\mathfrak{s}$ is an $\mathbb{F}_q[t]$-torsion point in $E_\mathfrak{s}(A)$ (resp. $\mathbf{v}_\mathfrak{s}$, $\mathbf{u}_\mathfrak{s}$ are $\mathbb{F}_q[t]$-linearly dependent in $E_\mathfrak{s}(A)$). In this talk, we are interested in the structure theory of the $t$-module $E_\mathfrak{s}(A)$. Poonen proved an analogue for Drinfeld modules of the Mordell-Weil theorem. We shall generalize his results to the case of specific families of $t$-modules. In particular, we prove that the $t$-module $E_\mathfrak{s}(A)$ is the direct sum of its torsion submodule, which is finite, with a free $\mathbb{F}_q[t]$-module of rank $\aleph_0$.

Oguz Gezmis    (National Tsing Hua University, Taiwan)

**Title:** Deformation of Multiple Zeta Values and Their Logarithmic Interpretation in Positive Characteristic

**Abstract:** Pellarin introduced the deformation of multiple zeta values of Thakur as elements over Tate algebras. In this talk, we discuss to relate these values to a certain coordinate of a higher dimensional Drinfeld module over Tate algebras. Furthermore we introduce the multiple polylogarithms and represent the deformation of multiple zeta values as a liner combination of multiple polylogarithms.

Henrik Bachmann     (Nagoya University, Japan)

**Title:** $q$-double zeta values and modular forms

**Abstract:** In their 2006 article, Gangl, Kaneko and Zagier established a duality between the space of even period polynomials of modular forms for $\mathrm{SL}_2(\mathbb{Z})$ and the space of formal double shuffle relations among double zeta values. We will discuss a variant of this duality for certain $q$-analogues of double zeta values. As one consequence, we obtain explicit rational solutions to the $q$-double shuffle equations in depth two generalizing Gangl–Kaneko–Zagier's "Bernoulli realization". This is joint work in progress with Ulf Kühn (Hamburg) and Nils Matthes (Oxford).

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## February 10. (Monday)

Tomoya Machide     (National Institute of Informatics, Japan)

**Title:** Computational results on ranks of matrices of extended double shuffle relations for modulo 2  (joint work with Tomohiro Sonobe)

**Abstract:** Extended double shuffle relations of multiple zeta values are linear relations with integer coefficients, which are obtained by shuffle and stuffle products and renormalization. It is conjectured that these relations suffice to give all linear relations for a fixed weight, and the conjecture is confirmed up to weight 20 by computer. In this talk, I introduce computational results which update record to weight 21. Our method uses ranks of matrices corresponding to these relations, where the entries of the matrices are taken modulo 2.

Yoshitaka Sasaki     (Osaka University of Health and Sport Sciences, Japan)

**Title:** The tangent symmetry for the multiple zeta-function

**Abstract:** Special values of the multiple zeta-function at non-positive integers depend on limiting processes except for some exceptions. Akiyama, Egami and Tanigawa, and Akiyama and Tanigawa introduced and evaluated multiple zeta values at non-positive integers for certain limiting processes, and showed relations among such multiple zeta values, called the tangent symmetry. In this talk, we show that the multiple zeta-function satisfies the similar symmetric relation. Then, tangent symmetries of multiple zeta values for several limiting processes are obtained via such formula.

Yuichiro Taguchi     (Tokyo Institute of Technology, Japan)

**Title:** Mordell-Weil groups over moderately large global fields

**Abstract:** A perfect field $K$ is said to be *Kummer-faithful* if, for any finite extension $L$ of $K$ and any semi-abelian variety $A$ over $L$, the Mordell-Weil group $A(L)$ has no non-trivial divisible subgroup. By the Mordell-Weil theorem, an algebraic number field of finite degree is Kummer-faithful. It is known that sub-$p$-adic fields are also Kummer-faithful. We construct a family of Kummer-faithful number fields which are not sub-$p$-adic. We also consider a function field analogue of this construction. This is a joint work with Yoshiyasu Ozeki.

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## February 11. (Tuesday)

Yoshihiro Takeyama     (University of Tsukuba, Japan)

**Title:** Finite multiple harmonic $q$-series at a root of unity and Kaneko-Zagier conjecture

**Abstract:** Kaneko and Zagier introduce two variants of multiple zeta value: the finite multiple zeta value (FMZV) and the symmetric multiple zeta value (SMZV). They conjecture that there exists a $\mathbb{Q}$-algebra isomorphism which sends FMZVs to SMZVs. In this talk we introduce a finite multiple harmonic $q$-series at a root of unity, which reproduces the FMZV and the SMZV through an algebraic operation and an analytic one, respectively. This property offers an explanation, though not a proof, for the conjecture due to Kaneko and Zagier. We also discuss recent progress in our framework. This talk is based on a joint work with H. Bachmann (Nagoya Univ.) and K. Tasaka (Aichi Prefectural Univ.).

Shuji Yamamoto     (Keio University, Japan)

**Title:** A variant of multiple polylogarithms and relations on them

**Abstract:** Let $\mathrm{Li}_{\mathbf{k}}(z_1, \ldots, z_r)$ be the multiple polylogarithm of shuffle type. We are mainly interested in the functions of one variable $z$ obtained by substituting 1 or $z$ to each $z_i$, and linear relations on them, such as $\mathrm{Li}_2(z) = \mathrm{Li}_{1,1}(1, z) - \mathrm{Li}_{1,1}(z, z)$. It turns out that some relations (duality, sum formula, ...) can be nicely formulated in terms of certain linear combinations of those polylogarithms. In this talk, we will introduce these functions and present known relations.

Ryota Umezawa    (Nagoya University, Japan)

**Title:** Relations between iterated log-sine integrals, multiple zeta values and multiple polylogarithms

**Abstract:** Iterated log-sine integrals introduced by the speaker are related to multiple zeta values and multiple polylogarithms. In this talk, we discuss relations between iterated log-sine integrals, multiple zeta values and multiple polylogarithms. In particular, we give an explicit evaluation of iterated log-sine integrals in terms of multiple zeta values and multiple polylogarithms as the main theorem. We also give some conjectures on iterated log-sine integrals and multiple zeta values.

Yoshinori Mishiba    (University of the Ryukyus, Japan)

**Title:** Logarithmic interpretation of Taylor coefficients of $t$-motivic multiple zeta values

**Abstract:** For each index, we construct a $t$-module such that the $\infty$-adic (resp. $v$-adic) multiple zeta value at the index appears in a certain coordinate of the $\infty$-adic (resp. $v$-adic) logarithmic vector of this $t$-module at a specific point, where $v$ is a finite place of a rational function field over a finite field. This is used to show the existence of a linear map from the space of $\infty$-adic MZV's to that of $v$-adic MZV's, which is a function field analogue of a conjecture of Furusho in characteristic zero. In $\infty$-adic setting, we give a generalization of this logarithmic interpretation. More precisely, we determine all of the coordinates of the logarithmic vector in terms of Taylor coefficients of ($\infty$-adic) $t$-motivic MZV's and $t$-motivic Carlitz multiple star polylogarithms. This is a joint work with Chieh-Yu Chang and Nathan Green.

Chieh-Yu Chang    (National Tsing Hua University, Taiwan)

**Title:** Algebra structure of multiple zeta values in positive characteristic

**Abstract:** In this talk, we will talk about MZV's over function fields in positive characteristic. We show that for each finite place $v$ of the base function field, the $v$-adic MZV's form an algebra with multiplication law given by Thakur's sum-shuffle product for $\infty$-adic MZV's. We then use Yu's sub-$t$-module theorem to show that there is an algebra homomorphism from $\infty$-adic MZV's to $v$-adic MZV's. Joint work with Yen-Tsung Chen and Yoshinori Mishiba.

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