Building-up Differential Homotopy Theory 2024 at Osaka

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Abstracts

Monday 4th of March

"***" indicates an on-line lecture.

Name: Shoji Yokura (Kagoshima University)
Title: On stratified spaces
Abstract: A stratified space is a topological space equipped with what is called stratification, which is a decomposition or partition of the space into disjoint subspaces called strata, satisfying some reasonable conditions. Another one is a poset-stratified space, which is a continuous function from a topological space to the Alexandrov space associated to a poset, a partially ordered set. In this talk I will discuss about relations between stratification and a poset-stratified space. If time permits, I will talk about some related topics as well.

Name: Paolo Giordano (University of Vienna)
Title: A conceptual introduction to Schwartz distributions and Colombeau generalized functions
Abstract: The need to describe abrupt changes or response of nonlinear systems to impulsive stimuli is ubiquitous in applications. But also within mathematics, L. Hörmander stated: "*In differential calculus one encounters immediately the unpleasant fact that not every function is differentiable. The purpose of distribution theory is to remedy this flaw; indeed, the space of distributions is essentially the smallest extension of the space of continuous functions where differentiability is always well defined*". We first describe the universal property of the space of distributions, but then we underscore the main deficiencies of this theory: we cannot evaluate a distribution at a point, we cannot make non-linear operations, let alone composition, we do not have a good integration theory, etc. We then present Colombeau theory of generalized functions, which is able to overcome several drawbacks of Schwartz distributions: pointwise evaluation, non-linear operations, partially defined composition, etc. Finally, we explain what are the limitations of Colombeau theory and where they originate from. This first talk aims to introduce the most well-known theories where continuous functions share several properties with smooth ones, a key step to build-up differential homotopy theory.

Name: Kazuhisa Shimakawa (Okayama University)
Title: Nonstandard diffeology and generalized functions
Abstract: In this talk I introduce a nonstandard extension (in the sense of A. Robinson) of the notion of diffeological spaces, and demonstrate its application to the theory of generalized functions. Just as diffeological spaces can be defined as concrete sheaves on the site consisting of Euclidean open sets and smooth maps, nonstandard diffeological spaces are defined as concrete sheaves on the site consisting of open subsets of nonstandard Euclidean spaces. By utilizing this similarity, we can show that nonstandard diffeological spaces form a super-category of the category of diffeological spaces which is self-enriched, complete, cocomplete and cartesian closed.

As a first application of our category, we show that the space of nonstandard functions is a smooth differential algebra over (a variant of) Robinson's field of nonstandard numbers and there is a linear injection of the differential vector space of Schwartz distributions into the differential algebra of nonstandard functions. In this regard, our algebra of nonstandard functions plays a role similar to Colombeau's algebra. But our algebra has an advantage over Colombeau's one in that it enables not only the multiplication of distributions but also the composition of them because it is a hom-object in a category.

P. Giordano has already constructed similar differential algebra by extending Colombeau's construction. Among other differences, the scalar of our theory is not Colombeau's ring, which is a ring with zero-divisors, but a (non-Archimedean) real closed field introduced by A. Robinson. This of course means that Colombeau's algebra cannot be embedded into our algebra. Still, it can be shown that there is a chain of algebra homomorphisms connecting the two algebras.

Name: Yukihiro Okamoto (Kyoto University)
Title: Construction of string homology of submanifolds by de Rham chains
Abstract: We fix a smooth manifold. For any compact smooth submanifold of codimension 2, Cieliebak-Ekholm-Latschev-Ng defined a chain complex whose homology they called the string homology. The construction involves a chain-level coproduct operation inspired by string topology, but it is restricted in lower degrees. They also proved that when the submanifold is a knot in $\mathbb{R}^3$, the zeroth degree part of the string homology is isomorphic to an algebraic invariant which derives from Floer theory in contact topology. In this talk, I will explain how to extend the string homology for submanifolds of arbitrary codimension, though the coefficient is reduced from the original one. The key point is that we use de Rham chains (instead of singular chains) of differentiable spaces of paths defined by Irie because they are suitable to define the chain-level operation in higher degrees. I will also show some examples of computations. If time permits, I will explain a perspective of connecting the string homology to Floer theory in contact topology.

Name: Jean-Pierre Magnot (University of Angers)
Title: On the geometry of equation manifolds ***
Abstract: The notion of equation manifold is an underlying topic in the theory of differential equations. As subsets of jet spaces, equation manifolds carry interesting geometric properties linked with geometric invariants of the underlying differential equation. In this talk, we will see how diffeologies can encompass the topological pathologies that one experiences, in particular when working on a non compact base space, then we will express globally some geometric properties that are only described on loci on a fixed equation manifold in the existing literature. We will finish with concerns on symmetries, currents, deformations, and the (diffeological) relationship between a wide class of equation manifolds with algebraic curves. This talk is based on a research program in progress, and we will try to include open directions in the exposition of the results recently published or pre-published.

Tuesday 5th of March

Name: Shun Wakatsuki (Nagoya University)
Title: DGA models in rational homotopy theory and their computation
Abstract: The differential graded algebra (DGA) of polynomial differential forms plays an important role in rational homotopy theory. But unfortunately they are too complicated to apply to explicit computation. This problem is resolved by using Sullivan algebras, which are much simpler and still contain enough information. In this talk, I will explain these algebras and show some example calculations. If time permits, computer-assited calculation will be also presented.

Name: Paolo Giordano (University of Vienna)
Title: How to deal with continuous functions as if they were smooth: Generalized smooth functions
Abstract: We present generalized smooth functions (GSF) theory, a nonlinear theory of generalized functions (GF) as informally used by physicists and engineers, where GF are ordinary set-theoretical maps defined on and taking values in a non-Archimedean ring (i.e. containing infinitesimal and infinite numbers) extending the real field (this problem has been faced e.g. by: Schwartz, Lojasiewicz, Laugwitz, Schmieden, Egorov, Robinson, Colombeau, Rosinger, Levi-Civita, Keisler, Connes, etc.); GSF are closed with respect to composition so that nonlinear operations are possible; these operations coincide with the usual ones for smooth functions; all classical theorems of differential and integral calculus hold; we have several types of sheaf properties, and GSF indeed form a Grothendieck topos; we have a full theory of ODE, and general existence theorems for nonlinear singular PDE, e.g. the Picard-Lindelöf theorem for PDE; every Cauchy problem with a smooth PDE is Hadamard well-posed; we can generalize the classical Fourier method also to non tempered GF (this problem has been faced e.g. by Gel'fand, Sobolev); we can also define generalized holomorphic functions, i.e. GF of a complex variable, a problem which is essentially impossible to solve without non-Archimedean mathematics; we have several applications in the calculus of variation with singular Lagrangians, lastoplasticity, general relativity, quantum mechanics, singular optics, impact mechanics (this problem has been faced by J. Marsden). The presentation will be mathematically rigorous, but we will also present several intuitive interpretations, which are useful to work with this theory.

Name: Atsushi Yamaguchi (Osaka Metropolitan University)
Title: Representations of groupoids in the category of plots
Abstract: For a Grothendieck site $(\mathcal{C}, J)$ and a functor F from $\mathcal{C}$ to the category of sets, we define a notion of "plots" which is a straightforward generalization of plots in diffeology. We denote by $\mathscr{P}_F(\mathcal{C}, J)$ the category of plots associated with $(\mathcal{C}, J)$ and F. In the case that $\mathcal{C}$ is a category of open sets of Euclidean spaces and smooth maps, J is a Grothendieck topology generated by open coverings of each objects of $\mathcal{C}$ and F is a forgetful functor from $\mathcal{C}$ to the category sets, $\mathscr{P}_F(\mathcal{C}, J)$ is nothing but the category of diffeological spaces and smooth maps. It can be shown that $\mathscr{P}_F(\mathcal{C}, J)$ is a quasi-topos, that is, $\mathscr{P}_F(\mathcal{C}, J)$ is (finitely) complete and cocomplete, locally cartesian closed and has a strong subobject classifier. We also observe that $\mathscr{P}_F(\mathcal{C}, J)$ is a bifibered category over the category of sets whose inverse image functor is defined from "induction" and direct image functor is defined from "subduction". By considering the fibered category of morphisms in $\mathscr{P}_F(\mathcal{C}, J)$, we define a notion of representations of groupoids in $\mathscr{P}_F(\mathcal{C}, J)$ and show the existence of induced representations.

Name: Enxin Wu (Shantou University)
Title: Some open questions related to diffeological vector spaces
Abstract: Diffeological vector spaces are vector spaces with extra smooth structures. They arise naturally from analysis, geometry, topology, etc. I will discuss some open questions related to this subject, including some background, their relationship with other fields, and current state of the art.

Name: Patrick Iglesias-Zemmour (The Hebrew University of Jerusalem)
Title: Example of Singular Reduction in Symplectic Diffeology
Abstract: I will present an example of symplectic reduction in diffeology where the space involved is infinite dimensional and when the reduction is singular. It is a mix of two extreme cases handled by diffeology. Precisely: The space in question is the space of periodic functions from the real to the complex numbers, equiped with a homogeneous symplectic structure. Then, we push this structure forward onto the space of Fourier coefficients. We shall give an autonomous description of the diffeology we obtain this way. Next, we equip the infinite product of tori with "tempered diffeology", introduced for the purpose of the example. Equipped with this diffeology, the infinite torus acts smoothly by automorphisms on the symplectic space of Fourier coefficients. We shall exhibit the moment map of the action of the infinite torus. Given a sequence of rationally independent real numbers we induce the real line in the infinite torus as an "irrational solenoid"and consider the induced action on the space of Fourier coefficients. This action is not free and generates infinitely many singular orbits. We consider then a generic level subspace of the moment map of the solenoid (they are all isomorphic to the infinite sphere) and we reduce it by the action of the solenoid. The quotient is then an "infinite quasi-projective space"cluttered by singular points corresponding to the singular orbits. We shall show finally that, in spite of the presence of singular orbits, the restriction of the symplectic form on the Hamiltonian level passes to the quotient and equips every infinite quasi-projective space with a "parasymplectic structure", that is, a closed 2-form. This would be also an opportunity to discuss the statute of quasifolds in infinite dimension since some infinite dimensional spaces, like this one, can play the role of "classifiants"for some kind of quasifolds. Actually, the example presented is a classifiant for quasi-spheres.

Wednesday 6th of March

Name: Kei Irie (Kyoto University)
Title: Chain level string topology and de Rham chains ***
Abstract: String topology is the study of intersection (co)products on loop spaces. I will explain a chain level construction of these products using the notion of de Rham chains. I will also mention an application of this chain level construction to symplectic topology.

Name: David Miyamoto (Max Planck Institute for Mathematics)
Title: Lie groupoids determined by their orbit spaces
Abstract: The orbit space of a Lie groupoid inherits a natural quotient diffeology. More generally, we have a quotient functor from the (Hilsum-Skandalis) category of Lie groupoids to the category of diffeological spaces. We introduce the notion of a lift-complete Lie groupoid, and show that the quotient functor restricts to an equivalence of the categories: of lift-complete Lie groupoids with isomorphism classes of submersive bibundles as arrows, and of quasi-étale diffeological spaces with local subductions as arrows. In particular, the Morita equivalence class of a lift-complete Lie groupoid is determined by its diffeological orbit space. Examples of lift-complete Lie groupoids include quasifold groupoids and étale holonomy groupoids of Riemannian foliations.

Thursday 7th of March

Name: Hiroshi Kihara (The University of Aizu)

Title: Categorical algebra of fiber bundles

Abstract: Fiber bundles are defined as locally trivial morphisms and play a central role in many geometries, with further structures such as vector bundle and principal bundle structures. On the other hand, diffeological fiber bundles, introduced by Iglesias-Zemmour, are defined by local triviality along plots and play an essential role in diffeology. We discuss these various types of fiber bundles in a general framework using the notions of Grothendieck topology and stack. This study is motivated by an attempt to understand twisted coefficients in diffeology and topology, which is related to a generalization of the de Rham theorem.

Name: Paolo Giordano (University of Vienna)

Title: Ideas about the Grothendieck topos of generalized smooth functions

Abstract: We start by presenting the sheaf property of generalized smooth functions (GSF), and trying to underscore the differences with respect to the classical sheaf property of smooth functions. We then show how GSF define the concrete site of gluable families and hence a related Grothendieck topos. The new sheaf property of GSF is important to show that hom-functors of gluable families are concrete sheaves over a concrete site. We then present ideas about several problems linking GSF theory and algebraic topology: how to extend manifolds (and diffeological spaces) with new non-Archimedean points, how to define generalized diffeological spaces and maps in the Grothendieck topos defined by gluable families, how to start synthetic differential geometry in these generalized spaces, how to define de Rham currents using GSF, how to start differential homotopy theory.