Building-up Differential Homotopy Theory 2023 at Aizu

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Abstracts

Sunday 5th of March

"***" indicates an on-line lecture.

Name: Yuzuru Inahama (Kyushu University)
Title: An Introduction to Rough Path Theory for Non-probabilists:
– Kuo-Tsai Chen meets Kiyosi Itô –

Abstract: In 1998, Terry Lyons invented a new theory that revolutionalized K. Itô's theory of stochastic integration and stochastic differential equation (SDE). It is now called rough path theory. In this theory, line integrals along a very bad continuous path are made possible. A typical example of such a path is a trajectories of Brownian motion, which controls SDEs.

The key of Lyons' idea is K. T. Chen's theory of iterated path integrals. By considering not just the path itself but also its iterated path integrals together, he was to able to generalize the theory of line integrals to a far-reaching extent. Such a pair of "a path and its iterated integrals" are called a rough path.

An integral equation based on line integrals is called a controlled ODE. A rough path version of a controlled ODE was naturally defined and is called rough differential equation (RDE). Note that this theory is deterministic, that is, it should be classified as a kind of real analysis, rather than probability theory.

But, it is also possible to take a Brownian rough path (i.e. a random rough path consists of iterated stochastic integral of Brownian motion) as a controlling rough path. Then, Itô's theory of stochastic integrals and SDEs are recovered. Moreover, the integration map and the solution map of a differential equation become everywhere-defined and continuous (this was formerly inconceivable). In other words, Itô's was made deterministic and was turned into real analysis.

In this talk, we will briefly explain the main storyline of this theory for non-probabilists.

Name: Keiichi Sakai (Shinshu University)
Title: The Fox-Hatcher cycle and the Vassiliev invariants
Abstract: It is known that the Vassiliev invariants for (long) knots can be described as the integrations over the configuration spaces associated with trivalent graph cocycles (R. Bott-C. Taubes, T. Kohno), while non-trivalent graph cocycles yield positive degree cocycles of the space of (long) knots (A. Cattaneo-P. Cotta-Ramusino-R. Longoni). The integrations of some concrete graph cocycles over the Gramain cycle and the Fox-Hatcher cycle are known to produce some Vassiliev invariants (V. Turchin, R. Longoni, K. Pelatt-D. Sinha, A. Mortier). I'd like to explain an approach to generalize these facts from the viewpoint of configuration space integrals. This talk is based on joint work with Saki Kanou that gives a Vassiliev invariants of order three.

Name: Hiroshi Kihara (The University of Aizu)
Title: De Rham calculus on diffeological spaces
Abstract: In this talk, we primarily discuss the de Rham theorem for diffeological spaces and its applications. First, we present the de Rham theorem along with its smooth homotopical variant. Second, we apply these results to orbifolds and infinite-dimensional smooth manifolds. We also discuss applications to the geometric realizations of (semi-)simplicial diffeological spaces, generalizing the works of Dupont and Bott-Shulman-Stasheff on the simplicial de Rham complexes of simplicial (finite-dimensional) smooth manifolds. If time permits, we would like to mention diffeological spaces for which the de Rham theorem do not hold.

Monday 6th of March

Name: Christian Blohmann (Max Planck Institute for Mathematics in Bonn)
Title: Cartan calculus on elastic diffeological spaces
Abstract: I will show that on elastic diffeological spaces there is a natural Cartan calculus consisting of vector fields and differential forms, together with the differential, inner derivative, and Lie derivative satisfying the usual bracket relations. The main idea is to define the Cartan calculus first on singular forms, which are the antisymmetric and multilinear functions on the powers of the tangent functor. In the second step, the condition of elasticity is used to show that these forms are isomorphic to those of the standard de Rham complex on diffeological spaces. This approach was motivated by applications to classical field theory and diffeological stacks, as I will explain at the end.

Name: Patrick Iglesias-Zemmour (The Hebrew University)
Title: Symplectic diffeology
Abstract: I will discuss some questions and results related to the extension of symplectic geometry to diffeology.

Name: Serap Gürer (Galatasaray University)
Title: Orbifolds as stratified diffeologies
Abstract: We will discuss some general properties of stratified spaces in diffeology. That leads to a general framework for the theory of stratifications. As an application, we consider the Klein stratification of diffeological orbifolds defined by the action of local diffeomorphisms. We show that it is a standard stratification in the sense that the partition of the space into orbits of local diffeomorphisms is locally finite (for orbifolds with locally finite atlases), it satisfies the frontier condition and the strata are locally closed manifold.

Tuesday 7th of March

Name: Yael Karshon (The University of Toronto, Tel-Aviv University)
Title: Smooth maps on convex sets
Abstract: There are several notions of a smooth map from a convex set to a Cartesian space Some of these notions coincide, but not all of them do. We construct a real-valued function on a convex subset of the plane that does not extend to a smooth function on any open neighbourhood of the convex set, but that for each k extends to a C^k function on an open neighbourhood of the convex set. It follows that the diffeological and Sikorski notions of smoothness on convex sets do not coincide. We show that, for a convex set that is locally closed, these notions do coincide. With the diffeological notion of smoothness for convex sets, we then show that the category of diffeological spaces is isomorphic to the category of so-called exhaustive Chen spaces. This is joint work with Jordan Watts.

Name: Toshitake Kohno (Meiji University, the university of Tokyo)
Title: Formal connections and the category of braid cobordisms
Abstract: The purpose of this talk is to describe an application of higher holonomy functors to the category of braid cobordisms. The theory of 2-connections on principal 2-bundles and their two dimensional holonomy has been developed by Baez, Schreiber and others. We obtain a categorical representation of the path 2-groupoid to the 2-Lie group. On the other hand, in the framework of the theory of iterated integrals, K.-T. Chen studied formal connections with values in the ring of non-commutative formal power series related to the cobar construction for the purpose of describing homology of loop spaces. We describe a universal form of two dimensional holonomy in terms of formal connections. We apply this method to the 2-category of braid cobordisms, where the 2-morphisms are surface spanning braids in 4-space. We obtain a power series of Lie type representation these 2-morphisms, which is considered as a 2-category version of the Kontsevich integral.

Name: Shintaro Kuroki (Okayama University of Science)
Title: Classification of locally standard torus actions
Abstract: An action of a torus T on a manifold M is called a locally standard if the stabilizer is a sub-torus and the non-zero isotropy weights are a basis to its weight lattice at each point. In this case, we can get the following three data:

1. the orbit space M/T which admits the structure of a smooth manifold with corners;
2. the label λ, called unimodular labeling, on the facets of M/T by the isotropy weights on the codimension 2 invariant submanifolds;
3. the degree 2 cohomology class c of M/T with the coefficients of the integral lattice of Lie algebra of T (this represents the "twistedness" of M over M/T).

In this talk, conversely, we show that three data recover the manifold M with the locally standard T-action up to equivariant diffeomorphism. This talk is the joint work with Yael Karshon.

Name: Urs Schreiber (New York University in Abu Dhabi)
Title: Introduction to Hypothesis H for Mathematicians ***
Abstract: The key open question of contemporary mathematical physics is the elucidation of the currently elusive fundamental laws of "non-perturbative" states —ranging from bound states as mundane as nucleons but more generally of quarks confined inside hadrons (declared a mathematical "Millennium Problem" by the Clay Math Institute), over topologically ordered quantum materials (currently sought by various laboratories), all the way to the ultimate goal of fundamentally understanding background-free quantum gravity and "grand unification". Now, the foremost non-perturbative effect in quantum physics is "flux quantization"; and I begin by explaining in detail how this finds its natural mathematical definition in differential cohesive homotopy theory. By going through key examples —as a fun exercise in cohesive homotopy theory — I explain how to systematically derive from this: 1. magnetic flux quantization (experimentally seen in superconductors), and then by just the same logic: 2. the widely expected Hypothesis K that "RR-flux" is quantized in topological K-theory, and in evident non-abelian generalization: 3. our novel Hypothesis H that "G-flux" is quantized in unstable Cohomotopy (i.e.: framed Cobordism). Depending on time and interest, I may close by indicating (A) how coupling to gravity enhances these flux quantization laws to *T*wisted & *E*quivariant & *D*ifferential (TED) refinements of these cohomology theories and (B) how Hypothesis H explains anyonic topological order controlled by KZ-monodromy in bundles of conformal blocks. This is joint work with Hisham Sati. Slides will be available at:
ncatlab.org/schreiber/show/Introduction+to+Hypothesis+H+for+Mathematicians

Name: Severin Bunk (The University of Oxford)
Title: Homotopy types of higher smooth spaces ***
Abstract: Differential homotopy theory refines ordinary homotopy theories of spaces. In particular, it encodes homotopical information using smooth, and possibly higher categorical, versions of spaces. To make a connection with classical homotopy theory, it is important to assign to any space in differential homotopy theory an 'underlying (ordinary) space', and this should be achieved in a homotopically meaningful way. In this talk I will demonstrate methods to establish this connection and, if time permits, use these to compute an example homotopy type of a smooth space which does not have an underlying set.

Wednesday 8th of March

Name: Takahito Naito (Nippon institute of technology)
Title: Cartan calculus in string topology
Abstract: The classical Cartan calculus for differential geometry consists of three types of derivations on the de Rham complex: the Lie derivative, the interior product and the exterior derivative. It is well known that these operations satisfy a relation called Cartan (magic) formula.

In this talk, I investegate a homotopy Cartan calculi in the sense of Fiorenza and Kowalzig on the free loop spaces. Roughly speaking, a homotopy Cartan calculus is a Cartan calculus which satisfies Cartan formula up to homotopy. I introduce a homotopy Cartan calculus on the Hochschild chain complex of the de Rham complex and give a geometric description of the calculus. Moreover, I also show that the description can be described by using the loop product and bracket in string topology. A part of this talk is based on joint work with K. Kuribayashi, S. Wakatsuki and T. Yamaguchi.

Name: Enxin Wu (Shantou University)
Title: Diffeological vector spaces ***
Abstract: I will present some old and new results related to diffeological vector spaces, focusing on the aspects of smooth homological algebra together with its connections to analysis, algebra, geometry and topology.