Building-up Differential Homotopy Theory

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Abstracts

Monday 4th of March

Name: **Patrick Iglesias-Zemmour** (Aix-Marseille Université & the Hebrew University)
Title: ***Inception of Diffeology***
Abstract:

I will summarise the beginning of diffeology when a diffeological space was still called an "espace différentiel". Then I'll describe the category Diffeology and its main properties. We shall review in details a few examples which emphasises the very special skills of diffeology, compared with other approaches.

Name: **Takahiro Matsuyuki** (Tokyo Institute of Technology)
Title: ***Moduli space of Chen's connections and characteristic classes***
Abstract:

For a manifold, we can regard the set of homotopy classes of Chen's connections as diffeological space through the diffeology of the de Rham complex. According to Chen's theorem, the quotient space of this space by the structure group plays a role as a classifying space of a smooth fiber bundle, and we can get characteristic classes of a fiber bundle through the cohomology of the space.

Name: **Akira Koyama** (Waseda Unversity)
Title: ***Homological properties of decomposition spaces***
Abstract:

Let X be a continuum in $\mathbb{R}^n$. It is known that the homotopy of the decomposition space $\mathbb{R}^n/X$ depends on only the shape type of X. In this talk we introduce an inverse sequence of compact ANR-sets whose limit has the same shape of $\mathbb{R}^n/X$. Using the inverse sequence, we construct a shape equivalence between the suspension $\Sigma(X)$ of X and $\mathbb{R}^n/X$. Then we investigate homologically local properties of the decomposition space $\mathbb{R}^n/X$ by certain continua such as the Case-Chamberlin curve C and the solenoids. As its consequence, we give an alternative fact that $\mathbb{R}^3/C$ has the trivial shape but $\mathbb{R}^3/\Sigma$ does not have.

Name: **Kazuhisa Shimakawa** (Okayama University)
Title: ***Generalized functions and Diffeology***
（一般関数とディフェオロジー）
Abstract:

We discuss a possibility of extending the definition of morphisms in the category of diffeological spaces. In particular, we aim at constructing a category of smooth spaces in which real functions on the Euclidean n-spaces are those generalized functions (including Schwartz distributions).
（微分空間のカテゴリーにおける射の定義の拡張について論じる。とくに，n 次元ユークリッド空間上の実数値関数の全体が（シュワルツ超関数を含む）一般関数の集合となるようなカテゴリーの構成を目指す）

Tuesday 5th of March

Name: **Tadayuki Haraguchi** (Naragakuen University)
Title: ***A homotopy structure of diffeological spaces***
（微分空間のホモトピー構造について）
Abstract:

In this talk, we introduce the ideas to construct a Quillen model structure on the category of diffeological spaces. We give a smooth cubical CW complexes corresponding to this model structure and discuss their homotopy extension properties.
（微分空間の圏上に Quillen モデル構造を導入するためのアイデアを紹介する。また、このモデル構造に対応する滑らかな CW 複体を与え、ホモトピー拡張性質に関係する内容を論ずる）

Name: **Jun Yoshida** (Tokyo Institute of Technology)
Title: ***Diffeological spaces in view of functors of points***
Abstract:

The notion of diffeological spaces introduced by Souriau generalizes manifolds. On the other hand, the theory of C^∞-rings is another attempt to the generalization. It is an extension of the theory of commutative rings, so we can consider the spectrum of each C^∞-rings. Although it originally arises from the logic, it recently turned out that the geometry on C^∞-rings provides several good aspects on homotopical problems of manifolds. For instance, Spivak, and later Joyce, introduced the notion of derived manifolds in terms of C^∞-rings so that we can consider homotpy intersections of submanifolds instead of actual intersections. This allows us to omit the transversality condition of two submanidols in the

study of intersections. Another advantage of the C^∞-geometry is that it contains several kinds of infinitesimals. As a result, we can discuss singularities in terms of morphisms of C^∞-rings. In this talk, I will begin with the observation that every Diffeological space gives rise to a C^∞-ringed space. As the geometry of C^∞-rings is very parallel to that of commutative rings, we will obtain a sheaf over "C^∞-Zariski site." In this point of view, we discuss fundamental groups and cohomologies of diffeological spaces.

Name: **Patrick Iglesias-Zemmour** (Aix-Marseille Université & the Hebrew University)
Title: ***Constructions in Diffeology I***
Abstract:

I will discuss the main constructions in diffeology developed after the founding papers, until today. That is: Higher homotopy theory, fiber bundles, various formulas and constructions in differential calculus, orbifolds, symplectic diffeology and moment map, symplectic reduction in presence of singularities. Then, if there is time I would discuss also the "Powerset Diffeology" and its applications to the structure of space of geodesic, and eventually, the connection between diffeology and Connes' non commutative geometry.

Name: **Serap Gürer** (Galatasaray University)
Title: ***Differential Forms on Stratified Spaces***
Abstract:

This is a joint work with Patrick Iglesias-Zemmour. In this talk, first, we extend the notion of stratified spaces to dieology. Then we characterise the subspace of stratified differential forms, or zero-perverse forms in the sense of Goresky−MacPherson, which can be extended smoothly into differential forms on the whole space. For that we introduce an index which outlines the behaviour of the perverse forms on the neighbourhood of the singular strata. We prove that, for cone-like stratified diffeological spaces, a zero-perverse form is the restriction of a global differential form if and only if its index is equal to one for every stratum.

Wednesday 6th of March

Name: **Shun Wakatsuki** (University of Tokyo)
Title: ***Gorenstein spaces and string topology***
Abstract:

A Gorenstein space is a generalization of a Poincaré duality space in view of singular cochain algebras. In string topology, we study the string operations, which are operations on the homology of the free loop space of a Gorenstein space. In this talk, we will introduce the notion of Gorenstein space and string operations, and explain an application of rational homotopy theory to them. Moreover, we will generalize string topology to mapping spaces from higher dimensional spheres using Gorenstein spaces essentially.

Name: **Patrick Iglesias-Zemmour** (Aix-Marseille Université & the Hebrew University)
Title: ***Constructions in Diffeology II***
Abstract:

I will discuss the main constructions in diffeology developed after the founding papers, until today. That is: Higher homotopy theory, fiber bundles, various formulas and constructions in differential calculus, orbifolds, symplectic diffeology and moment map, symplectic reduction in presence of singularities. Then, if there is time I would discuss also the "Powerset Diffeology" and its applications to the structure of space of geodesic, and eventually, the connection between diffeology and Connes' non commutative geometry.

Name: **Serap Gürer** (Galatasaray University)
Title: ***Differential Forms on Manifolds with Boundary and Corners***
Abstract:

This is a joint work with Patrick Iglesias-Zemmour. We identify the subcategory of manifolds with boundary and corners inside the category Diffeology. Then, we show that any differential form on a manifold with boundary or corners, embedded in a smooth manifold, extends smoothly into a differential form on an open neighbourhood.

Thursday 7th of March

Name: **Hiroshi Kihara** (University of Aizu)
Title: ***Smooth homotopy of infinite dimensional C^∞-manifolds***
Abstract:

The purpose of this talk is to apply homotopical algebra (or abstract homotopical methods) to global analysis on infinite dimensional C^∞-manifolds by embedding C^∞-manifolds into the category of diffeological spaces.

We begin by stating problems which we are concerned with.

Frölicher, Kriegl, and Michor [KM1] laid the foundation of infinite dimensional calculus, which is regarded as the top favorite of the final theory and called convenient calculus. However, one provides no efficient approach to one of the most important problems: to investigate how many smooth maps do there exist between given infinite dimensional C^∞-manifolds M and N.

Since the study of continuous maps between M and N are done by topological homotopy theory (or algebraic topology), we should formulate the problem as follows:

(a) When do the smooth homotopy classes of smooth maps between M and N bijectively correspond to the continuous homotopy classes of continuous maps ?

The following two problems are also important; Problem (b) is a generalization of Problem (a) and Problem (c) is closely related to Problems (a) and (b).

(b) Let $p : E \longrightarrow M$ be a smooth fiber bundle. When do the vertical smooth homotopy classes of smooth sections of E bijectively correspond to the vertical continuous homotopy classes of continuous sections ?

(c) Let G be a Lie group. When do the isomorphism classes of smooth principal G-bundles over M bijectively correspond to those of continuous principal G-bundles over M ? Let $\pi : P \longrightarrow M$ be a smooth principal G-bundle. When do the isotopy classes of smooth gauge transformations of P bijectively correspond to those of continuous gauge transformations of P ?

Problems (a), (b), and (c) are addressed by [KM2], [W2], and [MW, W1] respectively; roughly speaking, their answers state that the correspondences in question are bijective under the assumption of dim $M < \infty$ (, though Müller and Wockel ([MW], [W1], [W2]) work not in convenient calculus but in Keller's C^∞_c-theory). However, no essential answer is known in the case of dim $M = \infty$ since there exists no effective tool available in this case.

We develop smooth homotopy theory of diffeological spaces on the basis of the results of [K1] and apply it to solve such fundamental problems on C^∞-manifolds.

We introduce the basic notions on C^∞-manifolds and then state the main results.

A C^∞-manifold M is called *hereditarily C^∞-paracompact* if any open set U of M is C^∞-paracompact. A C^∞-manifold M is called *semiclassical* if M admits an atlas $\{(U_\alpha, u_\alpha)\}_{\alpha \in A}$ such that $u_\alpha(U_\alpha)$ and $u_\alpha(U_\alpha \cap U_\beta)$ are open in the model vector space E_α with respect to the locally convex topology for any $\alpha, \beta \in A$.

We show that if M is hereditarily C^∞-paracompact and semiclassical, then the correspondences in question are bijective; we actually give positive answers to higher homotopical versions of Problems (a)-(c) by observing that the relevant categories and functors can be enriched over the category of simplicial sets.

Many important C^∞-manifolds which appear in global analysis and algebraic topology are hereditarily C^∞-paracompact and semiclassical, and hence our results apply to them.

Name: **Patrick Iglesias-Zemmour** (Aix-Marseille Université & the Hebrew University)
Title: ***Constructions in Diffeology III***

Abstract:

I will discuss the main constructions in diffeology developed after the founding papers, until today. That is: Higher homotopy theory, fiber bundles, various formulas and constructions in differential calculus, orbifolds, symplectic diffeology and moment map, symplectic reduction in presence of singularities. Then, if there is time I would discuss also the "Powerset Diffeology" and its applications to the structure of space of geodesic, and eventually, the connection between diffeology and Connes' non commutative geometry.