Quantum White Noise Derivatives and Implementation Problem

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Quantum white noise calculus provides a systematic method for analysis of Boson Fock space operators, based on an infinite dimensional analogue of Schwartz type distribution theory. In this talk we discuss a new type of differential calculus for operators and its application to the implementation problem for CCR. This work is based on the long-term collaboration with Un Cig Ji (Chungbuk National University, Korea).

Let $H = L^2(T, dt)$, where T is a certain underlying space satisfying some natural conditions, and consider the Boson Fock space $\Gamma(H)$. Take suitable Gelfand triples

$$E \subset H \subset E^* \quad \text{and} \quad (E) \subset \Gamma(H) \subset (E)^*,$$

for the latter e.g., the Hida–Kubo–Takenaka space of white noise functions. It is essential in the quantum white noise calculus that the maps $t \mapsto a_t \in \mathcal{L}((E), (E))$ and $t \mapsto a_t^* \in \mathcal{L}((E)^*, (E)^*)$ are operator-valued test functions. Moreover, every white noise operator $\Xi \in \mathcal{L}((E), (E)^*)$ admits an infinite series expansion in terms of integral kernel operators and, therefore, is considered as a function of the quantum white noise $\Xi = \Xi(a_s, a_t^* \,;\, s, t \in T)$, for details see [4]. Then the derivatives

$$D_t^- \Xi = \frac{\delta \Xi}{\delta a_t} \quad \text{and} \quad D_t^+ \Xi = \frac{\delta \Xi}{\delta a_t^*}$$

are natural objects to discuss, where $\{a_s, a_t^* \,;\, s, t \in T\}$ plays a role of coordinate system.

In this talk we explain how the above mentioned derivatives are well formulated and applied to the implementation problem of CCR. Relevant results are reported in [3]. Another applications of quantum white noise derivatives are found in [1,2]

References

[1] U. C. Ji and N. Obata: *Quantum stochastic integral representations of Fock space operators*, Stochastics **81** (2009), 367-384.

[2] U. C. Ji and N. Obata: *Annihilation-derivative, creation-derivative and representation of quantum martingales*, Commun. Math. Phys. **286** (2009), 751–775.

[3] U. C. Ji and N. Obata: *Quantum white noise derivatives and associated differential equations for white noise operators*, preprint, 2009.

[4] N. Obata: "White Noise Calculus and Fock Space," Lect. Notes in Math. Vol. 1577, Springer, 1994.