

The Strange World of Non-amenable Symmetries

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Nonlinear sigma models with non-compact target space and non-amenable symmetry group were introduced long ago in the study of disordered electron systems. They also occur in dimensionally reduced quantum gravity; recently they have been considered in the context of the AdS/CFT correspondence. These models show spontaneous symmetry breaking in any dimension, even one and two (superficially in contradiction with the Mermin-Wagner theorem) as a consequence of the non-amenableity of their symmetry group. The low-dimensional models show other peculiarities: invariant observables remain dependent on boundary conditions in the Thermodynamic limit, the Osterwalder-Schrader reconstruction yields a non-separable Hilbert space and discontinuous representations occur. The ground state space, however, under quite general conditions, carries a unique unitary and continuous representation. The existence of a continuum limit in 2D is an open question: while the perturbative Renormalization Group suggests triviality, other arguments hint at the existence of a conformally invariant continuum limit at least for suitable observables.