Origin of ferromagnetism

— A "constructive condensed matter physics" approach —

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It is believed that strong ferromagnetic orders in some solids are generated by subtle interplay between quantum many-body effects and spin-independent Coulomb interactions between electrons. Here we describe our rigorous and constructive approach to ferromagnetism in the Hubbard model, which is a standard idealized model for strongly interacting electrons in a solid.

Our results can be classified into the following four classes.

The first is about our early results in the so-called flat band Hubbard model. Although the models are singular in the sense that the single-particle ground states are highly degenerate, the mechanism which generates ferromagnetism is clear and physically natural.

The second, which is probably the most important both physically and mathematically, is about our results for a class of non-singular Hubbard models. We introduce a class of Hubbard models in any dimensions which are nonsingular in the sense that both the Coulomb interaction and the density of states (at the Fermi level) are finite. We then prove that the ground states of the models exhibit saturated ferromagnetism, i.e., have maximum total spins. This provides nonsingular models of itinerant electrons with only spin-independent interactions where low energy behaviors are proved to be that of a "healthy" ferromagnetic insulator.

The third is our rather recent rigorous results on metallic ferromagnetism. We constructed the first rigorous examples of metallic ferromagnetism in the Hubbard models in dimensions higher than one.

The fourth is the latest results on the flat-band Hubbard model with magnetic flux. We discuss topological properties of the flat-bands and their relations to ferromagnetism.

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