Automorphic forms, trace formulas and zeta functions

RIMS Conference

Organizers: Yasuro Gon (Kyushu University)
Tomonori Moriyama (Osaka University)

Date :   January 17, 2011 – January 21, 2011
Venue :   Room 420
RIMS, Kyoto University, Japan

Titles and Abstracts

Jan.17 (Mon)

13:30-14:30 **Steven Spallone** (University of Oklahoma)

Title: *Stable trace formulas and discrete series multiplicities*

Abstract: Let G be a reductive algebraic group over $\mathbb{Q}$, and suppose that Γ is an arithmetic subgroup of $G(\mathbb{R})$ defined by congruence conditions. A basic problem in arithmetic is to determine the multiplicities of discrete series representations in $L^2(\Gamma\backslash G(\mathbb{R}))$, and in general to determine the traces of Hecke operators on these spaces. In this talk we give a conjectural formula for the traces of Hecke operators, in terms of stable distributions. It is based on a stable version of Arthur's formula for L^2-Lefschetz numbers due to Kottwitz. We mention evidence for this conjecture, first observed by Wakatsuki, in the case $G = GSp_4$ and $\Gamma = GSp_4(\mathbb{Z})$.

14:45-15:45 **Shin-ichi Kato** (Kyoto University), **Keiji Takano** (Akashi College of Technology)

Title: *Discrete series for symmetric spaces over p-adic fields*

Abstract: Various notions or theories concerning groups can be generalized (or "relativized") to those concerning symmetric spaces. Our work deals with several symmetric-space–counterparts of well-known theories concerning reductive p-adic groups. In our previous work, we have studied (i) Relative notion of cuspidality and a criterion in terms of Jacquet modules; (ii) Relative version of Jacquet's subrepresentation theorem. In this talk, we will present (iii) Relative notion of discrete series and a criterion in terms of exponents. We will give several concrete examples of relative discrete series.

16:00-17:00 **Masaaki Furusawa** (Osaka City University)

Title: *On the extension of the fundamental lemma for a certain relative trace formula to the full Hecke algebra*

Abstract: (This is a joint work with Kimball Martin and Joseph Shalika.) The speaker and Martin formulated a certain relative trace formula for the group $GSp(4)$, which we expect to establish an explicit formula for the central critical value of the spinor L-function in terms of the Bessel period. We proved the fundamental lemma for the unit element of the Hecke algebra. In this talk, we would like to discuss the recent result of the extension of the fundamental lemma to the full Hecke algebra by the speaker, Martin and Shalika. Our method is by the Fourier inversion based on the computations of the Plancherel measures utilizing the explicit formulas for the Bessel and Whittaker models. The theory of the Macdonald polynomials plays an important role.

Jan.18 (Tue)

9:45-10:45 **Sho Takemori** (Kyoto University)

Title: *p-adic Siegel-Eisenstein seires of degree 2*

Abstract: I will introduce an explicit formula for Fourier coefficients of Siegel-Eisenstein series of degree two with a primitive character of any conductor. Moreover, I will show that there exists a p-adic analytic family which consists of Siegel-Eisenstein series of degree two and a certain p-adic limit of Siegel-Eisenstein series of degree two is actually a Siegel-Eisenstein series of degree two.

11:10-12:00 **Kenichi Namikawa** (Osaka University)

Title: *On special values of adjoint L-functions and the congruence of modular forms*

Abstract: Around 1980, K. Doi and H. Hida found a meaning of the special value of certain degree 3 L-functions, so called adjoint L-functions of cusp forms. They discovered that if a prime divides "algebraic part" of the adjoint L-function of a cusp form, the prime is a congruence prime for the cusp form. E. Ghate and M. Dimitrov proved analogues of Hida's theorem in Hilbert modular case. E. Urban also proved a similar result in case of cusp forms on $GL(2)$ over imaginary quadratic fields. In this talk, we prove such a result in case of cusp forms on $GL(2)$ over number fields.

13:30-14:30 **Gombodorj Bayarmagnai** (University of Tokyo)

Title: *Differential equations satisfied by principal series Whittaker functions on $SU(2,2)$*

Abstract: In this talk, we consider principal series representations of $SU(2,2)$ with higher dimensional minimal K-type and discuss about a system of differential equations for Whittaker functions associated with these K-types.

14:45-15:45 **Tadashi Miyazaki** (Tokyo University of Agriculture and Technology)

Title: *Explicit formulas of principal series Whittaker functions on $Sp(2,\mathbb{C})$*

Abstract: In this talk, we give explicit formulas of principal series Whittaker functions on $Sp(2,\mathbb{C})$. We also report their applications to the computation of Novodvorsky's archimedean zeta integrals.

16:00-17:00 **Takahiro Hayata** (Yamagata University), **Harutaka Koseki** (Mie University), **Takayuki Oda** (University of Tokyo)

Title: *Toward explicit formula of the reproducing kernel of the large discrete series of $SU(3,1)$*

Abstract: The group $G = SU(3,1)$ has the large discrete series representations. We compute the radial part of the reproducing kernel of these representations. The difficulty of computation is that the maximal compact subgroup $K = U(3)$ of G is rather big. Though the real rank of G is one, the computation, which employs a system of the Dirac-Schimd equations, is rather involved, but has a beautiful symmetry.

Jan.19 (Wed)

9:45-10:45 **David Yuen** (Lake Forest College)

Title: *Paramodular Forms*

Abstract: We discuss techniques for computing paramodular forms and the data from such computations for low weights and arbitrary levels. For prime levels, we use the method of integral closure to compute the space of paramodular cusp forms and we discuss the results for weight 2 and prime levels less than 600. For arbitrary levels, we discuss the technique of restriction to Jacobi forms to compute the space of paramodular cusp forms and we discuss the results for weight 2 and all levels up to 1000. We discuss the following applications: (1) This gives substantial evidence for the Paramodular Conjecture, which is the genus 2 analog of the Modularity Theorem in genus 1. (2) We see examples of twists of paramodular forms.

11:00-12:00 **Takeo Okazaki** (Nara Women's University)

Title: *Theta correspondence for $GSp(4)$ and $GSO(4,2)$*

Abstract: van Geemen and van Straten conjectured a spinor L-function of a Siegel modular cusp form of degree 2 of weight 3 is a Rankin-Selberg convolution of L-functions of elliptic modular cusp forms. We verify this conjecture by a theta correspondence for $GSp(4)$ and $GSO(4,2)$.

13:30-14:30 **Shuichi Hayashida** (Osaka University)

Title: *Lifting of pairs of elliptic modular forms to Siegel modular forms of half-integral weight of degree two*

Abstract: The aim of this talk is to show a lifting from two elliptic modular forms to a Siegel modular form of half-integral weight of degree 2. More precisely, if k is an even integer, then one can obtain a lifting from two elliptic modular forms of weight $2k-2$ and $2k-4$ to a Siegel modular form of weight $k-\frac{1}{2}$ of degree 2. The existence of this lifting had been conjectured by T. Ibukiyama and the speaker through numerical examination of Euler factors.

14:45-15:45 **Kaoru Hiraga** (Kyoto University), **Tamotsu Ikeda** (Kyoto University)

Title: *On the trace formula for the covering groups of SL_2 and its application*

Abstract: We discuss a stabilization of the trace formula for the covering groups of SL_2. As an application, we construct a Kohnen plus space for Hilbert modular forms of half-integral weight.

16:00-16:30 **Yasutaka Ihara** (Kyoto University)

Title: *Saito-Shintani lifting and "Fermat Last Theorem" (historical comment)*

Abstract: (to the memory of late Hiroshi Saito)
A basic result of Langlands-Tunnel referred to and used in Wiles-Taylor proof of Shimura-Taniyama conjecture asserts that any absolutely irreducible "odd" two dimensional Galois representation over $\mathbb{Q}$ with finite [solvable] image is modular. (Note that "any solvable" is somewhat misleading, because the only possible solvable image modulo center is cyclic, dihedral, A_4 or S_4, and Langlands(-Tunnel) new contribution is for the latter two cases. As is well-known, the last S_4 case is used by Wiles to lift a mod 3 Galois representation.). cf. R. Langlands "Base Change for $GL(2)$" (1980) (Preprint in summer 1975, with a subtitle "Saito-Shintani lifting ⋯"). As the above subtitle shows, the first crucial contribution to this line of development was due to Hiroshi Saito (his thesis), which was soon improved by T. Shintani. Their works had been presented in the US-Japan Seminar at Michigan in June 1975 which fired Langlands interest and led very shortly to the above amazing improvement that includes the weight-one case. In this talk, I shall recall briefly this line of development initiated by H. Saito, which should not be forgotten and which we can be proud of.

16:40-17:00 **Orginizers**

Title: *On the next RIMS conference*

18:00-20:00 **Banquet**

at *Camphora*

Jan.20 (Thu)

9:45-10:45 **Tadahisa Nara** (Tohoku University)
Title: *On the Mordell-Weil group of the elliptic curve $y^2 = x^3 + n$*

Abstract: We study an infinite family of the elliptic curves in the form $y^2 = x^3 + n$ over $\mathbb{Q}$ with three explicit integral points. The points are independent in certain cases. In this talk we describe how to compute bounds of the canonical heights of the points. Using the result we show that any pair in the three points can always be a part of a basis of the free part of the Mordell- Weil group. This is joint work with Yasutsugu Fujita.

11:00-12:00 **Dan Yasaki** (University of North Carolina at Greensboro)

Title: *On modular forms and elliptic curves over the field of fifth roots of unity*

Abstract: In this talk, we describe some recent results of our computational investigation of modular forms over the cyclotomic field of fifth roots of unity. We discuss the methods used as well as connections with elliptic curves. This is joint work with Paul Gunnells and Farshid Hajir.

13:30-14:30 **Yiannis Petridis** (University College London)

Title: *Dissolving cusp forms into resonances: Higher order Fermi's Golden Rules*

Abstract: For a hyperbolic surface the spectral decomposition consists of continuous bands (provided by Eisenstein series) and possible embedded eigenvalues and is still not completely understood. Closely associated with the Eisenstein series are the resonances. For $SL(2, \mathbb{Z})$ there are plenty of embedded eigenvalues (Maass cusp forms). The embedded eigenvalues are in general unstable and tend to become resonances. The sufficient dissolving condition was identified by Phillips and Sarnak and is elegantly expressed in Fermi's Golden Rule. Their work is a first order approximation. We prove formulas for all higher approximations and obtain necessary and sufficient conditions for dissolving a cusp form into a resonance. We relate the result to the special values of L-series involving a Rankin-Selberg convolution of the cusp form with higher order automorphic forms. This is joint work with Morten S. Risager.

14:45-15:45 **Yoshinori Mizuno** (Tokushima University)

Title: *Dirichlet series associated with square of the class numbers*

Abstract: We give a meromorphic continuation and a functional equation for Mellin transforms and Rankin-Selberg convolutions of Maass forms of half-integral weight satisfying the plus condition. As an application, we study a certain Dirichlet series associated with square of the class numbers of binary quadratic forms. The result implies a suitable definition of Koecher-Maass series associated with non-holomorphic Siegel-Eisenstein series of degree two

and weight two.

16:00-17:00 **Masao Tsuzuki** (Sophia University)

Title: *Spectral means of central values of automorphic L-functions for $GL(2)$*

Abstract: Starting with Green's functions on adele points of $GL(2)$ considered over a totally real number field, we elaborate an explicit version of the relative trace formula, whose spectral side encodes informaton on period integrals of cuspidal waveforms along a maximal split torus. As an application, we prove two kinds of asymptotic mean formula for certain central L-values attached to cuspidal waveforms with square-free level

Jan.21 (Fri)

9:45-10:45 **Bernhard Heim** (German University of Technology in Oman)
Title: *Igusa's modular form of weight* 10 *(joint work with Atsushi Murase (Kyoto Sangyo University))*

Abstract: The Igusa modular form of weight 10 is the first Siegel cusp form of degree 2 with respect to the full modular group. In this talk I would like to address certain properties and indicate a program towards a characterization of this form by additive and multiplicative symmetries.

10:55-11:55 **Satoshi Wakatsuki** (Kanazawa University)

Title: *Congruences modulo 2 for dimensions of spaces of cusp forms*

Abstract: In this talk, we give some congruences modulo 2 for dimensions of spaces of Siegel cusp forms of degree one or two. First, we review some known results for congruences between dimensions of spaces of cusp forms and class numbers of imaginary quadratic fields. Next, we give our main result, which is a congruence relation for the degree two case. It is related to Arthur's conjecture for $PGSp(2)$.