Symposium on Arithmetic Geometry

Kyushu University, Fukuoka, JAPAN

Oct 19 - Oct 21, 2012

Naoki Imai (Kyoto Univ)

Title: Potentially good reduction loci of Shimura varieties
Abstract: We give a definition of a potentially good reduction locus of a pre-abelian Shimura variety. Its cohomology can be regarded as a variant of the nearby cycle cohomology. We compare the cohomology of this locus with the cohomology of the Shimura variety. This is a joint work with Yoichi Mieda.

Alan Lauder (University of Oxford, United Kingdom)

Title: Computations with classical and p-adic modular forms
Abstract: I will present p-adic algorithms for computing Hecke polynomials and Hecke eigenforms associated to spaces of classical modular forms using the theory of overconvergent modular forms. The algorithms have a running time which grows linearly with the logarithm of the weight and are well suited to investigating the dimension variation of certain p-adically defined spaces of classical modular forms. Implementations are now available in the distributed versions of the computer algebra systems MAGMA and SAGE (as well as on my homepage).

Iku Nakamura (Hokkaido Univ)

Titles: A net of closed immersions of the moduli $SQ^{\mathrm{toric}}_{g,K}$ of TSQASes into Alexeev's complete moduli $\overline{A}_{g,d}$
Abstract: There are two relevant moduli spaces, our second geometric compactification $SQ^{\mathrm{toric}}_{g,K}$ of the moduli of abelian varieties [N], and Alexeev's complete moduli $\overline{A}_{g,d}$ of generalized abelian varieties, each coupled with a semiabelian group action and an ample divisor [A]. We prove that if $|K| = d^2$, there is a $(d-1)$-dimensional effective family over $\mathbb{Z}[\zeta_d, 1/d]$ of *closed immersions* of $SQ^{\mathrm{toric}}_{g,K}$ into $\overline{A}_{g,d}$, whose image is Zariski open in $\overline{A}_{g,d}$.
[A] V. Alexeev, Complete moduli in the presence of semiabelian group action, Ann Math (2), 155 (2002), 611-708
[N] I. Nakamura, Another canonical compactification of the moduli space of abelian varieties, in *Algebraic and Arithmetic Structures of Moduli Spaces*, Advanced Studies of Pure Mathematics 58, (2010) 69-135

Kentaro Nakamura (Hokkaido Univ)

Titles: A generalization of Kato's local epsilon conjecture for (ϕ, Γ)-modules over the Robba ring I, II

Abstract: In his preprint 'Lectures on the approach to Iwasawa theory of Hasse-Weil L-functions via BdR, Part II', Kazuya Kato proposed a conjecture which is called local epsilon conjecture. This conjecture roughly says that the determinant of the Galois cohomology of a family of p-adic Galois representations has a canonical base which can be described by using Deligne-Langlands epsilon constants. In our talks, we generalize his conjecture for families of (ϕ, Γ)-modules over the Robba ring and prove a part of this conjecture in some special cases (more precisely, trianguline case). The two key ingredients are the recent result of Kedlaya-Pottharst-Xiao on the fininiteness of cohomologies of (ϕ, Γ)-modules and my result on Bloch-Kato exponential maps for (ϕ, Γ)-modules.

Tadashi Ochiai (Osaka Univ)

Title: Coleman map for families of ordinary Galois representations

Abstract: For a given family of ordinary p-adic Galois representation, Coleman map is an interpolation of Kato's dual exponential map. When we have a suitable Euler system on a given family of ordinary p-adic Galois representation, the Coleman map serve as a machinary which transforms the Euler system to a p-adic L-function and it gives an important step to prove the Iwasawa Main Conjecture in each situation. After stating general conjectural pictures on the existence of Euler system and Coleman map, I will state some results related to the context for some deformations constructed by Hida theory.

Shun Ohkubo (Univ of Tokyo)

Title: On differential modules associated to de Rham representations in the imperfect residue field case

Abstract: Let K be a CDVF of mixed characteristic $(0, p)$, whose residue field admits a finite p-basis. Denote the absolute Galois group of K by G. For a given de Rham representation V of G, we will construct a differential module $N_{\mathrm{dR}}(V)$, which is a generalization of Laurent Berger 's $N_{\mathrm{dR}}(V)$ in the perfect residue field case. We also explain some properties of this differential module.

Denis Osipov (Steklov Institute of Mathematics, Russia)

Title: Noncommutative reciprocity laws and non-ramified two-dimensional Langlands correspondence

Abstract: We will describe the local non-ramified Langlands correspondence for two-dimensional local fields (following an approach of M. Kapranov). It will be constructed the categorical analog of principal series representations of general linear groups over two-dimensional local fields. The main ingredients of the contrsuction are some central extensions of these groups (these groups are defined over two-dimensional local fields or over adelic rings of two-dimensional arithmetic schemes). We will prove reciprocity laws for these central extensions, i.e. splittings of these central extensions over some subgroups defined over semilocal rings constructed by means of points and one-dimensional subschemes of a two-dimensional arithmetic scheme.

Title: Two-dimensional harmonic analysis and the Riemann-Roch theorem for algebraic surfaces over finite fields

Abstract: This is a survey talk on joint papers with A.N. Parshin, how to construct the main ingredients of harmonic analysis for adelic rings of two-dimensional arithmetic schemes (the main difficulty is that this adelic ring is not locally compact). The application of the theory is a new proof of the Riemann-Roch theorem for algebraic surfaces over finite fields, using the analogs of Poisson formulas like the well-known proof for algebraic curves by means of usual harmonic analysis. The references are arXiv:0707.1766v3 [math.AG], arXiv:0912.1577v2 [math.AG] and arXiv:1107.0408v2 [math.AG].

Atsushi Shiho (Univ of Tokyo)

Title: Parabolic log convergent isocrystals

Abstract: For a smooth variety X over a perfect field k of characteristic $p > 0$, Katz and Crew proved the equivalence between the category of p-adic representations of the fundamental group of X and the category of unit-root convergent F-isocrystals on X. In this talk, we prove a log version of the above equivalence: for a pair (X, Z) of a smooth variety X and a simple normal crossing divisor Z, the category of p-adic representations of the log fundamental group of (X, Z) is equivalent to the category of generically semistable adjusted parabolic log convergent F-isocrystals on X of slope 0. When (X, Z) is a log curve satisfying a certain strong liftability condition, the above categories are further equivalent to the category of generically semistable parabolic F-vector bundles on certain log rigid analytic curve. This is a kind of p-adic analogue of a result of Mehta-Seshadri and conjectured by Weng.

Atsushi Shiho (Univ of Tokyo)

Title: On a generalization of local Ogus-Vologodsky correspondence

Abstract: Given a smooth scheme over $\mathbb{Z}/p\mathbb{Z}$ with a lift of relative Frobenius to $\mathbb{Z}/p^2\mathbb{Z}$, Ogus and Vologodsky proved an equivalence of the category of quasi-nilpotent modules with integrable connections and that of quasi-nilpotent Higgs modules. In this talk, we consider the case of a smooth scheme over $\mathbb{Z}/p^n\mathbb{Z}$ with a lift of relative Frobenius to $\mathbb{Z}/p^{n+1}\mathbb{Z}$ and prove a certain generalization of their result.

Kotaro Sugahara (Kyushu Univ)

Title: Weng's Zeta Functions for Curves over Finite Fields and the Counting Miracle

Abstract: Recently, L. Weng and D. Zagier prove that non-abelian zeta functions for elliptic curves over finite fields satisfy the Riemann hypothesis. In their proof, the special and general counting miracles play a key role. These are the relations between the so-called alpha and beta invariants, defined using global sections and automorphism groups of semi-stable bundles. In this talk, we will review the theory of Weng's zetas and establish a general counting miracle for curves of any genus. Our method, motivated by the study of quivers, is very different.

Takahiro Tsushima (Kyushu Univ)

Title: On reductions of some affinoids in the Lubin-Tate space in ramified case

Abstract: Recently, Jared Weinstein computes the reductions of some affinoids in the Lubin-Tate space for GL_h using perfectoid space due to Peter Scholze. The reductions realize the local Langlands correspondence (LLC) for unramified representations.

In this talk, we find affinoids and compute their reductions, whose cohomology realizes ramified representations with low conductors. If the residue characteristic of the base field of the Lubin-Tate space divides h, a Galois representation, which is not essentially tame in a sense of Bushnell-Henniart, does exist. We give some geometric realization of such Galois representation with low conductor. This is a joint work with Naoki Imai.

Cristian Virdol (Kyushu Univ)

Title: On the cyclicity of finite abelian varieties

Abstract: Let A be an abelian variety of dimension r, defined over a number field F. For $\mathfrak{p}$ a finite prime of F, denote by $F_{\mathfrak{p}}$ the residue field at $\mathfrak{p}$. If A has good reduction at $\mathfrak{p}$, let $\bar{A}$ be the reduction of A at $\mathfrak{p}$. In my talk, under GRH, I will prove an asymptotic formula for the number of primes $\mathfrak{p}$ of F, with $N_{F/Q}\mathfrak{p} \leq x$, for which $\bar{A}(F_{\mathfrak{p}})$ has at most $2r-1$ cyclic components.

Yi Zhang (Fudan Univ, China)

Title: Smooth toroidal compactification of Siegel variety form view of Kähler-Einstein metric

Abstract: We will talk about Kähler-Einstein metrics on Siegel varieties through compactifications. We get a global volume formula. Using it, we show that the Kähler-Einstein metric on $\mathcal{A}_{g,\Gamma}(g > 1)$ endows certain restraint combinatorial conditions for all toroidal smooth compactifications of $\mathcal{A}_{g,\Gamma}$ with normal crossing boundary divisor. We also study the asymptotic behaviour of logarithmical canonical line bundles on smooth toroidal compactifications of $\mathcal{A}_{g,\Gamma}$, and find that they degenerate sharply, despite of being big and nef.