Intersection Homology of Moduli Spaces of Semi-Stable Arithmetic G-Torsors

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Abstract

Naturally associated to a split reductive group defined over a number field are the (total) moduli spaces of the associated arithmetic torsors and their semi-stable companions. In this article, we formulate some conjectures to determine the intersection homology of the semi-stable moduli spaces in terms of the intersection homology of the reduced Borel-Serre compactifications $/L^2$-cohomology of the total moduli spaces.

1 Intersection Homology and L^2-Cohomology for Moduli Spaces of G-Torsors

Let F be a number field with $\mathcal{O}_F$ its ring of integers. Denote by S, resp. S_{fin}, resp. S_∞, the set of inequivalent normalized places, resp. finite places, resp. infinite places, of F. For each $v \in S$, denote by F_v the v-completion of F. As usual, denote the adelic ring of F by $\mathbb{A}$ so that $\mathbb{A} = \prod'_{v\in S_{\mathrm{fin}}} F_v \times \prod_{\sigma\in S_\infty} F_\sigma$.

Let G be a split reductive group over F. Fix a maximal compact subgroup K of $G(\mathbb{A})$. By [7], the quotient space $G(F)\backslash G(\mathbb{A})/K$ parametrizes the moduli space of arithmetic G-torsors on the arithmetic curve $\overline{\mathrm{Spec}\,\mathcal{O}_F}$. In particular, if we fix a slope μ and consider only the moduli space $\mathcal{M}_{F,G}^{\mathrm{tot}}(\mu)$ of arithmetic G-torsors of slope μ on $\overline{\mathrm{Spec}\,\mathcal{O}_F}$. Then it can be parametrized by $G(F)Z_{G^1(\mathbb{A})}\backslash G^1(\mathbb{A})/K$.[1] Denote by $\mathcal{M}_{F,G}^{\mathrm{ss}}(\mu)$ the moduli space of semi-stable arithmetic G-torsors of slope μ on $\overline{\mathrm{Spec}\,\mathcal{O}_F}$.

Fix a minimal parabolic subgroup P_0 and a maximal split torus T of G contained in P_0. Denote by $(V, \langle\ ,\ \rangle; \Phi, \Phi^\pm; \Delta; W; \rho)$ be the root system associated to (G, P_0, T). In particular, $\rho := \dfrac{1}{2}\displaystyle\sum_{\alpha\in\Phi^+}\alpha$. Moreover, for a standard parabolic subgroup P of G, denote the induced data by $(V_P, \langle\ ,\ \rangle_P; \Phi,_P \Phi_P^\pm; \Delta_P; W_P; \rho_P)$, even it may not form a root system.

As usual, for a piecewise-linear pseudomanifold X, define its Poincare polynomial $IP_X(t)$ for the associated intersection homology by

$$IP_X(t) := \sum_{i=0}^{\dim_{\mathbb{R}} X} \dim_{\mathbb{R}} IH_i(X)_{\mathbb{R}}\, t^i.$$

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[1]Here for the definition of $G^1(A)$ and the unknown related notations in this section, please refer to any standard reference on automorphic representations, or better, [7].

Here, $IH_i(X)_{\mathbb{R}}$ denotes the i-th intersection homology group of X and $IH_i(X)_{\mathbb{R}} := IH_i(X) \otimes \mathbb{R}$.

Conjecture 1. *For each proper standard parabolic subgroup P of G, there is a rational function $h_{\Delta_P,\rho_P}(t)$ depending only on $\langle \rho_P, \alpha^\vee \rangle$ $(\alpha \in \Delta_P)$ such that*

$$IP_{\mathcal{M}^{\mathrm{ss}}_{F,G}(\mu)}(t) = \sum_P (-1)^{\dim(\mathfrak{a}_P^G)}\, h_{\Delta_P,\rho_P}(t) \cdot IP_{\mathcal{M}^{\mathrm{tot},*}_{F,M_P}(\mu)}(t). \tag{1}$$

Here P runs through all standard parabolic subgroups of G, M_P denotes the standard Levi subgroup of P, and $\mathcal{M}^{\mathrm{tot},*}_{F,M_P}(\mu)$ denotes the reduced Borel-Serre compactification of $\mathcal{M}^{\mathrm{tot}}_{F,M_P}(\mu)$.

For example, if $G = \mathrm{SL}_n$, the standard parabolic subgroups $P = P_{n_1,n_2,\ldots,n_k}$ are determined uniquely by the ordered partition $n = n_1 + n_1 + \ldots + n_l$. In this case $h_{\Delta_{P_{n_1,\ldots,n_k}},\rho_{P_{n_1,\ldots,n_k}}}(t)$ is a rational polynomial depending only on $n_1 + n_2, n_2 + n_3, \ldots, n_{k-1} + n_k$.

Furthermore, note that being symmetric domains, there is a natural L^2-cohomology theory for the moduli space $\mathcal{M}^{\mathrm{tot}}_{F,G}(\mu)$. Recall that when the symmetric domains are of hermitian type, Zucker conjectured and Looijenga and Saper-Stein proved that the L^2-cohomology groups of these domains are coincide wth the intersection cohomology groups of the Satake, Baily-Borel compactifications of these domains. We expect this holds in general if we instead using the so-called reduced Borel-Serre compactifications.

Using the Poincare duality for the intersection cohomology on the compatified spaces, we may equally calculate the intersection homology of these spaces in terms of the L^2-cohomology of the symmetric domains. Denote the associated Poincare polynomial by $P^{(2)}_{\mathcal{M}^{\mathrm{tot}}_{F,G}(\mu)}(t)$.

Conjecture 2. *For each proper standard parabolic subgroup P of G, there is a rational function $h_{\Delta_P,\rho_P}(t)$ depending only on $\langle \rho_P, \alpha^\vee \rangle$ such that*

$$IP_{\mathcal{M}^{\mathrm{ss}}_{F,G}(\mu)}(t) = \sum_P (-1)^{\dim(\mathfrak{a}_P^G)}\, h_{\Delta_P,\rho_P}(t) \cdot P^{(2)}_{\mathcal{M}^{\mathrm{tot}}_{F,M_P}(\mu)}(t). \tag{2}$$

Here P runs through all standard parabolic subgroups of G, M_P denotes the standard Levi subgroup of P.

Recall that, by Grothendieck's theorem on reductive schemes in SGA 3[3], there is a isogeny between M_P and the product of its simple factors $M_{P,1}, M_{P,2}, \ldots, M_{P,\ell_P}$ (counting with multiplicities). Based on this, we introduce the following

Conjecture 3. *For each proper standard parabolic subgroup P of G, there is a rational function $h_{\Delta_P,\rho_P}(t)$ depending only on $\langle \rho_P, \alpha^\vee \rangle$ such that*

$$\begin{aligned} IP_{\mathcal{M}^{\mathrm{ss}}_{F,G}(\mu)}(t) &= \sum_P (-1)^{\dim(\mathfrak{a}_P^G)}\, h_{\Delta_P,\rho_P}(t) \cdot \prod_{i=1}^{\ell_P} IP_{\mathcal{M}^{\mathrm{tot},*}_{F,M_{P,i}}(\mu)}(t) \\ &= \sum_P (-1)^{\dim(\mathfrak{a}_P^G)}\, h_{\Delta_P,\rho_P}(t) \cdot \prod_{i=1}^{\ell_P} P^{(2)}_{\mathcal{M}^{\mathrm{tot}}_{F,M_{P,i}}(\mu)}(t). \end{aligned} \tag{3}$$

A Intersection Homology

In this section, we briefly recall some basic definitions, useful constructions, and fundamental properties for intersection homology introduced by Goresky-MacPherson. The main reference is [**?**].

A.1 Definition

By definition, a para-compact Hausdorff topological space X is called a *topological stratified space*, if there exists a filtration

$$X = X_m \supseteq X_{m-1} \supseteq \cdots \supseteq X_1 \supseteq X_0 \tag{4}$$

of closed subsets X_j of X satisfies the following condition.

> For every point $x \in X_j \setminus X_{j-1}$, there exists a neighborhood N_x of x in X, a compact $(m-j-1)$-dimensional topologically stratified space L with filtration
>
> $$L = L_{m-j-1} \supseteq \cdots \supseteq L_1 \supseteq L_0,$$
>
> and a homeomorphism
>
> $$\varphi : N_x \longrightarrow \mathbb{R}^j \times C(L)$$
>
> where $C(L)$ denotes the open cone on the space L, such that, homeomorphically,
>
> (1) $\varphi : N_x \bigcap X_j \stackrel{\simeq}{\longrightarrow} \mathbb{R}^j \times \big\{\text{Vertex of } C(L)\big\}$.
>
> (2) $\varphi : N_x \bigcap X_{j+i+1} \stackrel{\simeq}{\longrightarrow} \mathbb{R}^j \times C(L_i)$ if $m - j - 1 \geq i \geq 0$.

It is easy to see that, for a fixed j, $X_j \setminus X_{j-1}$ is a topological manifold of dimensional j. The connected components of these manifolds are called the *strata of* X. Up to homeomorphism, L is uniquely determined by the stratum in which x lies. Conversely, x is called the *link* of the stratum.

A para-compact Hausdorff topological space X of dimension m is called a *topological pseudomanifold* if X possesses a topological stratification satisfying the following conditions

(1) $X \setminus X_{m-1}$ is dense in X, and

(2) $X_{m-1} = X_{m-2}$.

And it is said to be *irreducible* if $X \setminus X_{m-1}$ is connected. In this case, $H_m(X, \mathbb{Z})$ is either $\mathbb{Z}$ or 0. If it is $\mathbb{Z}$, a choice of generator for $H_m(X, \mathbb{Z})$ is called an *orientation* of X and X is called *orientable*.

Recall that, by definition, an *n-simples* σ in $\mathbb{R}^d$ is the convex hull of a set of points $v_0, v_1, \ldots, v_n$ such that $v_1 - v_0, v_2 - v_0, \ldots, v_n - v_0$ are linearly independent. The *vertices*, resp. the *dimension*, resp. the *faces*, of σ are defined to be the points $v_0, v_1, \ldots, v_n$, resp. n, resp. the $(n-1)$-simplices whose vertices are contained in $\{v_0, v_1, \ldots, v_n\}$. A set N of simplices is called a *simplicial complex* in $\mathbb{R}^d$ such that

(1) If $\sigma \in N$, so is each faces of σ.

(2) If $\sigma, \tau \in N$ and $\sigma \cap \tau \neq \emptyset$, then $\sigma \cap \tau$ is a simplex whose vertices are contained in the common vertices of σ and τ.

(3) If $x \in \sigma \in N$, then there is a neighborhood $\mathfrak{U}$ of x in $\mathbb{R}^n$ such that $\mathfrak{U} \cap \tau \neq \emptyset$ for only finitely many simplices $\tau \in N$.

In this case, we set the *support* of N by

$$|N| := \bigcup_{\sigma \in N} \sigma.$$

In addition, a *trangulation of a topological manifold* X is a homeomorphism $T : |N| \to X$.

A topological pseudomanifold X is called a *piecewise-linear* if X admits a triangulation $T : |N| \to X$ which is compatible with the filtration in the sense that each X_j is a union of simplices. Denote by $C_i^T(X)$ be the space of all finite simplicial i-chains of X with respect to T with natural boundary map $\partial : C_i^T(X) \to C_{i-1}^T(X)$. By definition, the *support* $|\xi|$ of a simplical i-chain $\xi = \sum_{\sigma \in N^{(i)}} \xi_s \sigma$ is given by

$$|\xi| := \bigcup_{\xi_\sigma \neq 0} T(\sigma).$$

Recall that a *perversity* is a function $p : \{2, 3, \ldots, m\} \to \mathbb{N}$ such that

(1) $p(2) = 0$.

(2) $p(i+1) = p(i)$ or $p(i) + 1$ for all $2 \leq i \leq m-1$.

By definition, an i-chain $\xi \in C_i^T(X)$ is called *p-allowable* if

$$\dim_{\mathbb{R}} |\xi| \cap X_{m-k} \leq i - k + p(k) \qquad (\forall\, i \geq -\infty)$$

Let $I^p C_i^T(X)$ be the subspace of $C_i^T(X)$ consisting of all p-allowable i-chains ξ in $C_i^T(X)$ whose faces $\partial\xi$ are p-allowable $(i-1)$-chains.

Note that if T' is a refinement of the triangulation T, then the natural inclusion map $C_i^T(X) \to C_i^{T'}(X)$ sends a chain ξ to a chain with the same support and hence, via restrictions, induces a natural map $I^p C_i^T(X) \to I^p C_i^{T'}(X)$. Therefore, it makes sense to define the space $I^p C_i(X)$ of *piece-linear intersection i-chain* as the limit of the $I^p C_i^T(X)$'s over all triangulations T of X which are compatible with the stratification. Naturally induced from the boundary maps ∂ is the boundary map

$$\partial : I^p C_i(X) \to I^p C_{i-1}(X).$$

By definition, the *i-th intersection homology group $I^p H_i(X)$ of X with perversity* p is given by the quotient group

$$I^p H_i(X) := \frac{\operatorname{Ker} \partial : I^p C_i(X) \to I^p C_{i-1}(X)}{\operatorname{Im} \partial : I^p C_{i+1}(X) \to I^p C_i(X)}.$$

Similarly, we can introduce the homology group $I^p H_i^T(X)$, and the intersection cohomology groups $I^p H^i(X)$ and $I^p H_T^i(X)$. In addition, we denote $IH_i^{\mathrm{cl}}(X)$ the intersection homology groups with closed support. For later use, we set,

$$IC_i(X) := I^m C_i(X) \qquad \text{and} \qquad IH_i(X) := I^m H_i(X),$$

where $m := \dim_{\mathbb{R}} X$. Obviously, if X is compact, we have

$$IH_i(X) \simeq IH_i^{\mathrm{cl}}(X).$$

Without using triangulation, following King, we may proceed as follows. Assume that X is a topological m-pseudomanifold equipped with a stratification 4. Let Δ_i be a standard i-simplex in $\mathbb{R}^{i+1}$. Then the set of its j-subsimplices is called the *j-skeleton of* Δ_i, and a singular i-simplex in X defined by a continuous map $\sigma : \Delta_i \to X$ is called *p-allowable* if, for $k \geq 2$

$$\sigma^{-1}(X_{m-k} \smallsetminus X_{m-k-1}) \subseteq (i - k + p(k))\text{-skeleton of } \Delta_i.$$

It is not too difficult to check that a singular i-chain is p-allowable if it is a formal linear combination of p-allowable singular simplices. Introduce the subspace $I^pS_i(X)$ of $S_i(X)$ to be the collection of p-allowable chains satisfying the condition that its boundaries are all p-allowable and define the i-th perversity p singular intersection homology group $I^pH_i^{\mathrm{sin}}(X)$ of X (and the $I^pH_{i,c}^{\mathrm{sin}}(X)$with closed supports by considering p-allowable locally finite singular chains) similarly.

Proposition 4 (King)**.** *If X is a piecewise-linear pseudomanifold, then*

$$I^pH_i^{\mathrm{sin}}(X) \simeq I^pH_i(X) \qquad (\forall\, i).$$

A.2 Basic Properties

A.2.1 Functoriality

By definition, a continuous map $f : X \to Y$ between topological stratified spaces is *stratum-preserving* if the inverse image of each stratum of Y is a union of strata of X, or equivalently, if the image of each stratum of X is contained in a single stratum of Y. Moreover, a stratum-preserving map $f : X \to Y$ is called *placid*, resp. *codimension-preserving*, if for each stratum of Y, the codimension of its inverse image in X is bigger or equal to, resp. equal to, the codimension of the stratum in X. In addition, two codimension-preserving maps $f : X \to Y$ and $g : Y \to X$ are said to be *stratum-preserving homotopy equivalent* if $g \circ f : X \to X$ and $f \circ g : Y \to Y$ are homotopic to the identity maps Id_X and Id_Y via the codimension-preserving maps $\mathrm{pr}_1 : X \times [0,1] \to X$ and $\mathrm{pr}_1 : Y \times [0,1] \to Y$, respectively.

Proposition 5. *Let $f : X \to Y$ be continuous map between topological pseudomanifolds. Then f induces isomorphisms*

$$f_* : I^pH_i(X) \simeq I^pH_i(Y).$$

provided that one of the following two conditions is satisfied.

(1) f is a homeomorphism.

(2) (Freidman) f is a stratum-preserving homotopy equivalence.

A.2.2 Poincare Duality

Theorem 6 (Goresky-MacPherson)**.** *Let X be an oriented topological psudomanifold of dimension m. If p and q are complementary perversities in the sense that $p(i) + q(i) = i - 2$ for each i, then there are non-degenerate bilinear forms*

$$I^pH_i(X) \times I^qH_i^{\mathrm{cl}}(X) \to \mathbb{Q}.$$

A.2.3 Relative Homology Groups and Mayer-Vietoris Exact Sequence

Let X be a topological psudomanifold, and let U be an open subset of X. Composition with the natural inclusion $U \hookrightarrow X$ yields a natural inclusions $I^pS_i(U) \hookrightarrow I^pS_i(X)$ which commute with the boundary maps. We introduce the induced quotient group

$$I^pS_*(X,U) := \frac{I^pS_*(X)}{I^pS_*(U)}$$

and call them the *relative intersection homology groups of the pair* (X,U).

Proposition 7 (Goresky-MacPherson). *Let X be a topological psudomanifold and let U be an open subset of X. Then*

(1) There exists a long exact sequence

$$\ldots \to I^pH_*(X) \to I^pH_*(X,U) \to I^pH_{*-1}(U) \to \ldots$$

(2) Let A be a closed subset of U such that $X \smallsetminus A$ is topological pseudomanifold. Then there are natural iusomorphisms

$$I^pH_i(X,U) \simeq I^pH_i(X \smallsetminus A, U \smallsetminus A).$$

(3) mathrm(Mayer-Vietoris Exact Sequence) Let V be am open subset of X. Then there is a natural long exact sequence

$$\ldots \to IH_i(U) \oplus IH_*(V) \to IH_*(X) \to IH_{*-1}(U \cap V) \to \ldots$$

A.2.4 Künneth Theorem

Proposition 8 (King). *Let X be a topological psudomanifold. Then there are natural isomorphisms*

$$I^pH_i(X \times (0,1)) \simeq I^pH_i(X).$$

A.2.5 Examples

Example 1. Let X be a topological pseudomanifold.

(1) If X is a topological manifold, then there are natural isomorphisms

$$I^pH_i(X) \simeq J_i(X).$$

(2) If X is irreducible, of even dimension, and admits only an isolated singularities x, then for the lower middle perversity p given by $i \mapsto [(i-2)/2]-1$, we have

$$I^pH_i(X) = \begin{cases} H_i(X) & i > \frac{1}{2}\dim_{\mathbb{R}} X \\ \mathrm{Im}(H_i(X \smallsetminus \{x\}) \to H_i(X)) & i > \frac{1}{2}\dim_{\mathbb{R}} X \\ H_i(X \smallsetminus \{x\}) & i < \frac{1}{2}\dim_{\mathbb{R}} X \end{cases}$$

(3) If $(X, \partial X)$ is an even dimensional manifold with boundary. Then, for the lower middle perversity p, we have

$$I^p H_i(X \cup_{\partial X} C(\partial X) = \begin{cases} H_i(X, \partial X) & i > \frac{1}{2}\dim_{\mathbb{R}} X \\ \mathrm{Im}(H_i(X) \to H_i(X, \partial X)) & i > \frac{1}{2}\dim_{\mathbb{R}} X \\ H_i(X) & i < \frac{1}{2}\dim_{\mathbb{R}} X \end{cases}$$

Here, the *(open) cone* $C(L)$ on a compact Hausdorff topological space is defined as the topological space $L \times [0,1)$ by dentifying $L \times \{0\}$ to a single point (called the vertex of the cone).

Example 2 (Cone)**.** Let X be a compacxt topological pseudomanifold of dimension $m \geq v1$. Then we have

$$I^p H_i(C(X)) \simeq \begin{cases} I^p H_i(X) & i < m - p(m+1) \\ 0 & otherwise \end{cases}$$

and

$$I^p H_i(C(X), C(X) \smallsetminus \{v\}) \simeq \begin{cases} 0 & i \leq m - p(m+1) \\ I^p H_{i-1}(X) & otherwise \end{cases}$$

Example 3 (Normalizations)**.** Let X be a m-dmiensonal topological psudomanifold with filtration (4). Then X is called (topological) *normal* if for each $x \in X$ there exists an open neighborhood U of x in X such that $U \smallsetminus X_{m-2}$ is connected. Every yopological pseudomanifold admits a *normalization* $\pi : \widetilde{X} \to X$ in which $\widetilde{X}$ is normal and π is a homeomorphism onto X_{m-2}. By a result of Goresky-MacPherson, we have

(1) If X is normal, there are canonical isomorphisms

$$I^{\mathrm{top}} H_i(X) \simeq H_i(X) \qquad \text{and} \qquad I^{\mathrm{zero}} H_i(X) \simeq H^{m-i}(X).$$

Here, top, resp. zero, denotes the top perversity, resp. the zero perversity, defined by $j \mapsto j-2$, resp. by $j \mapsto 0$.

(2) If $\pi : \widetilde{X} \to X$ is a normalization of X, then there are canonical isomorphisms

$$I^p H_i(\widetilde{X}) \simeq I^p H_i(X).$$

Example 4. Let X be a quasi-projective algebraic variety of pure dimension n. A filtration

$$X = X_n \supseteq X_{n-1} \supseteq \ldots \supseteq X_0 \tag{5}$$

is called *Whitney stratification* of X if, for each j,

(1) X_j is a closed subvarity of X

(2) $X_j \smallsetminus X_{j-1}$ is either empty or a non-singular quasi-projective subvariety of pure dimension j.

(3) The strata S_α of the stratification, i.e. the connected components S_α of $X_j \smallsetminus X_{j-1}$, should satisfy the following Whitney conditions.

(a) If a sequence of points $\{x_i\} \subset S_\alpha$ converges to a point $x \in S_\beta$, then the tangent space $T_x S_\beta$ of S_β at x should be contained in the limit of $\{T_{x_i} S_\alpha\}$ whenever the limit exists.

(b) If two sequences $\{x_i\} \subset S_\alpha$ and $\{y_i\} \subset S_\beta$ converge to the same point, then the limit of the sequence of lines $\{\overline{x_i y_i}\}$ is contained in the limit of the tangent spaces $\{T_{x_i} S_\alpha\}$ whenever the both limits exist.

Following Whitney, every quasi-projective algebraic variety of pure dimension admits a Whitney stratification. Moreover, Borel gives a method to ensure that every such a Whitney stratification (5) transforms X into a topological pseudomanifold (of dimension $2n$) characterized by the filtration

$$X = X'_{2n} \supseteq X'_{2n-1} \supseteq \ldots \supseteq X'_0$$

where $X'_{2i} = X'_{2i-1} = X_i$. In addition, as proved by Lojasiewicz and Goresky, X admits a triangulation which is compatible with the filtration.

Based from the above discussion, it makes sense to talk about the intersection homology for quasi-projective algebraic varieties. Standard properties such as the Poincare duality, the Lefschetz hyperplane theorem, the hard Lefschetz trheorem, the Hodge decomposition and Hodge signature theorem have their companions in this framework. In addition, if $\widetilde{X}$ is the normalization of X in the sense of algebraic geometry, then

$$IH_i(X) \simeq H_i(\widetilde{X}).$$

We leave the details to the reader.

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