Geometric and Harmonic Analysis on Homogeneous Spaces and Applications

Sousse, December 12-16, 2011

Abstracts

Ahmad Abouelaz: Nazarov's Inequality on the Lattice $\mathbb{Z}^n$.

In this talk, we prove the following theorem: Let S be a finite subset in $\mathbb{Z}^n$ and Σ be a subset in $\left(-\frac{1}{2},\frac{1}{2}\right)^n$. Then for all $f \in l^1\left(\mathbb{Z}^n\right)$, we have

$$\begin{aligned}\sum_{m\in\mathbb{Z}^n} |f(m)|^2 \;\leq\; \gamma(S,\Sigma)\{ &\sum_{m\in\mathbb{Z}^n\setminus S} |f(m)|^2 + \int_{\mathbb{T}^n\setminus\Sigma} |\mathcal{F}f(x)|^2\,dx \\ &+ \int_{\Sigma} |\mathcal{F}f(x)|^2 \left(1-\prod_{i=1}^{n}\left(\frac{\sin(\pi x_i)}{\pi x_i}\right)^2\right) dx,\}\end{aligned}$$

where $\gamma(S,\Sigma)$ is the Nazarov 's constant and $\mathcal{F}f$ is the discrete Fourier transform given by $\mathcal{F}f(x) = \sum_{m\in\mathbb{Z}^n} f(m)\exp(-2i\pi xm)$, $\forall x \in \mathbb{T}^n$.

As a consequence, we get the following result: let S be a finite subset in $\mathbb{Z}^n$ and Σ be a subset in $\left(-\frac{1}{2},\frac{1}{2}\right)^n$. Then for all $f \in L^1(\mathbb{T}^n)\cap L^2(\mathbb{T}^n)$

$$\begin{aligned}\int_{\mathbb{T}^n} |f(x)|^2\,dx \;\leq\; \gamma(S,\Sigma)\{ &\sum_{m\in\mathbb{Z}^n\setminus S} \left|\widehat{f}(m)\right|^2 + \int_{\mathbb{T}^n\setminus\Sigma} |f(x)|^2\,dx \;+ \\ &+ \int_{\Sigma} |f(x)|^2 \left(1-\prod_{i=1}^{n}\left(\frac{\sin(\pi x_i)}{\pi x_i}\right)^2\right) dx\},\end{aligned}$$

where $\widehat{f}(m) = \int_{\mathbb{T}^n} f(x)\exp(-2i\pi mx)\,dx$.

We also prove other results on the Radon transform on the torus, in addition we show the Mandelbrojt 's theorem (Uncertainty principle) on the lattice $\mathbb{Z}^n$. (This is a joint work with T. Kawazoe).

Mouadh Akriche: Real algebraic surfaces fibered in curves of genus 2 (The classification of singular fibers).

Our aim is to classify the fibers of real smooth algebraic surfaces fibered by curves of genus 2. As a first step, we explain how one can reduce the situation by using the

real stable fibration associated to these surfaces of general type. Then, We discuss the geometry (and topology) of (general) singular fibers of elliptic monodromy.

Walid Aloulou: Pre-Gerstenhaber algebras up to homotopy.

In this lecture, we study the concepts of algebra up to homotopy for a structure defined by two operations $\cdot$ and $[,]$. Having determined the structure of Gerstenhaber algebras up to homotopy (G_∞ algebra), we generalize this construction and we define a structure of pre-Gerstenhaber algebra up to homotopy (pre G_∞ algebra).

Given a structure of Zinbiel and differential graded pre-Lie algebras and working over the corresponding operads, we will give an explicit construction of the associate pre-Gerstenhaber algebra up to homotopy. This is a joint work with Didier Arnal and Ridha Chatbouri.

Didier Arnal: Canonical Coordinates for a class of solvable groups.

For class R, type I solvable groups of the form NH, N nilpotent, H abelian, we construct an explicit layering with cross-sections for coadjoint orbits. We show that any ultrafine layer has a natural structure of fiber bundle. The description of this structure allows us to build explicit local canonical coordinates on the layer. Joint work with Bradely Currey and Bechir Dali.

Salma Azaouzi: A generalized analogue of Hardy's uncertainty principle on compact extensions of $\mathbb{R}^n$.

Let K be a compact subgroup of automorphisms of $\mathbb{R}^n$. We prove in this paper a generalized analogue of Hardy's uncertainty principle on the semi-direct product $K \ltimes \mathbb{R}^n$ extending earlier partial results on Euclidean motion groups where $K = SO(n)$. As a consequence, a complete analogue of classical Hardy's theorem is obtained. The representation theory and the Plancherel formula play an important role in the proofs. This is a joint work with Mounir Elloumi and Ali Baklouti.

Mabrouk Ben Ammar: Deformation of $\mathfrak{sl}(2)$ and $\mathfrak{osp}(1|2)$-Modules of Symbols.

We consider the $\mathfrak{sl}(2)$-module structure on the spaces of symbols of differential operators acting on the spaces of weighted densities. We compute the necessary and sufficient integrability conditions of a given infinitesimal deformation of this structure and we

prove that any formal deformation is equivalent to its infinitesimal part. We study also the super analogue of this problem getting the same results. This is a joint work with Imed Basdouri.

Salem Bensaîd: From the trigonometric to the rational Dunkl setting.

I will summarize the main results obtained with B. Orsted on this topic. The relationship with spherical functions on Riemannian and pseudo-Riemannian symmetric spaces will be elucidated.

Abdellatif Bentaleb: On extension and refinement of the Poincaré inequality.

The aim of this paper is to analyze the heat semigroup $(\mathcal{N}_t)_{t\geq 0}$ generated by the usual Laplacian operator Δ on $\mathbb{R}^d$ equipped with the $d-$dimensional measure Lebesgue. We obtain and study, via a method involving some semigroup techniques, a large family of functional inequalities that does not exist in the literature and with the local Poincaré and reverse local Poincaré inequalities as particular cases. As an consequence, we establish in parallel a new functional and integral inequalities related to the Hermite semigroup.

Michel Duflo: Frobenius Lie subalgebras of simple Lie algebras.

A Frobenius Lie algebra is a Lie algebra for which the coadjoint action has an open orbit. I present results on Frobenius Lie subalgebras of a simple complex Lie algebra which contain a Cartan subalgebra, with a special emphasis on the "Ooms spectrum" (the Ooms spectrum is an equivalent for Frobenius Lie algebras of the set of exponents of a simple Lie algebra). This is a joint work with M. S. Khalgui and P. Torasso.

Saîd Fahlaoui: Sharp estimates for some inequalities related to the ultraspherical semigroup.

This paper studies the ultraspherical semigroup $(\exp(tL^{(\alpha)}))_{t\geq 0}$, $\alpha > 0$, generated by the operator $L^{\alpha}f(x) := (1-x^2)f'' - \alpha x f'$ acting on the Hilbert space $\mathbb{L}^2((-1,+1), \mu_\alpha)$ with weight the ultraspherical probability measure $\mu_\alpha(dx) = c_\alpha(1-x^2)^{\frac{\alpha}{2}-1}dx$. We obtain new inequalities which interpolates in a sharp wary between the Poincaré inequality and

the logarithmic Sobolev inequality. We also prove a extension and refinement of the Poincaré and Jensen inequalities.

Jacques Faraut: Analysis on symmetric cones and multivariate Meixner-Pollaczek polynomials.

We consider the homogeneous space $\Omega = G/K$ associated to a Gelfand pair (G, K), where G is a Lie group, and K a compact subgroup. The algebra $\mathscr{D}(\Omega)$ of G-invariant differential operators on Ω is commutative. Fix a $\mathcal{C}^\infty$ and K-invariant function ψ_0 on Ω such that, for every $D \in \mathscr{D}(\Omega)$, $D\psi_0 \in L^2(\Omega)$. Define the linear form ℓ on $\mathscr{D}(\Omega)$ by

$$\ell(D) = (D\psi_0 \mid \psi_0)_{L^2(\Omega)},$$

and the inner product

$$\langle D_1, D_2 \rangle = \ell(D_1 D_2^*).$$

We are interested in the construction of an orthogonal basis in $\mathscr{D}(\Omega)$. In the case of a symmetric cone Ω, and choosing $\psi_0(u) = e^{-\mathrm{tr}u}$, the multivariate Meixner-Pollaczek polynomials provide such an orthogonal basis. Joint work with Masato Wakayama.

Mahmoud Filali: Approximable interpolation sets.

In a group, I_0-sets and Sidon sets are interpolation sets for the almost periodic functions and the Fourier-Stieltjes functions, respectively. Both of these sets have been extensively studied.

Extending and unifying these concepts, we introduce in this talk the notion of approximable interpolation sets for general algebras of functions on a locally compact group. We characterize the approximable interpolation sets for for the right uniformly continuous functions and for the weakly almost periodic functions. Joint work with Jorge Galindo.

Ahmad Fitouhi: On some $q-$integral transforms and applications.

We study in this talk some $q-$integral transforms and some of their applications in $q-$analysis.

Hidenori Fujiwara: Plancherel formula for the restriction of unitary representations of nilpotent Lie groups.

Let $G = \exp \mathfrak{g}$ be a connected and simply connected nilpotent Lie group with Lie algebra g. Let π be an irreducible unitary representation of G, $K = \exp \mathfrak{k}$ an analytic

subgroup of G, $\hat{K}$ the unitary dual of K and $p : \mathfrak{g}^* \to \mathfrak{k}^*$ the restriction mapping. Let Ω be the coadjoint orbit of G associated with π, $\tilde{\mu}$ a finite measure on Ω equivalent to the $G-$invariant measure on Ω and $\theta : \mathfrak{k}^* \to \hat{K}$ the Kirillov mapping for K. Now we regard $\tilde{\mu}$ as a measure on $\mathfrak{g}^*$, take the image measure $\mu = (\theta \circ p)_*(\tilde{\mu})$ on $\hat{K}$ and denote $m(\sigma)$ for $\sigma \in \hat{K}$, the number of $K-$orbits contained in $p^{-1}(\theta^{-1}(\sigma)) \cap \Omega$. Then the restriction $\pi|K$ to K decomposes into irreducibles as follows:

$$\pi|K \simeq \int_{\hat{K}}^{\oplus} m(\sigma)\sigma d\mu(\sigma).$$

According to this decomposition and by use of intertwining distributions, we try to describe a Plancherel formula for the restriction $\pi|K$. (Joint work with Ali Baklouti and Jean Ludwig)

Abdelhamid Hassairi: Cauchy-Stieltjes Kernel Families with cubic pseudo-variance functions.

We extend the study of Cauchy-Stieltjes kernel families to cover generating measures with support that is unbounded on one side. This extension leads to the characterization of a class of cubic pseudo-variance functions corresponding to free-infinitely divisible distributions without the first moment. A joint work with Wlodzimierz Bryc, University of Cincinnati, USA.

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Junko Inoue: The norm of the L^p-Fourier transform on compact extensions of $\mathbb{R}^n$.

This is a joint work with Ali Baklouti. Let G be a unimodular Lie group, $\widehat{G}$ the unitary dual of G, $1 < p \leq 2$ and $q = p/(p-1)$. We are concerned with the L^p-Fourier transform $\mathcal{F}^p(G) : L^p(G) \to L^q(\widehat{G})$. When the group G is a compact extension of $\mathbb{R}^n$, we show that the norm of the L^p-Fourier transform on G is equal to that on $\mathbb{R}^n$. That is, $\|\mathcal{F}^p(G)\| = A_p^n$, where $A_p = (p^{1/p}/q^{1/q})^{1/2}$.

Hideyuki Ishi: The Wallach set for a homogeneous bounded domain.

The Wallach set is defined for a symmetric bounded domain as the set of parameters admitting the analytic continuation of holomorphic discrete series representations. In this talk, we generalize the definition for a homogeneous bounded domain, and discuss the unitary representation corresponding to the parameter in the Wallach set. Recent results about the isotropy subgroup will play an important role for the observation of the representation.

Imed Kédim: On the rigidity Conjecture of deformations on solvable Lie groups.

Let G be an exponential solvable Lie group and H a connected Lie subgroup of G. Given any discontinuous subgroup Γ for the homogeneous space $\mathscr{M} = G/H$ and any deformation of Γ, the deformed discrete subgroup may utterly destroy its proper discontinuous action on $\mathscr{M}$ as H is not compact (except for the case when it is trivial). To emphasize this specific issue, we present in this talk different questions related to the geometry of the parameter and the deformation spaces of any discrete subgroup Γ acting properly discontinuously and fixed point freely on G/H for an arbitrary H. We focus attention of some rigidity conjectures substantiated in the more general context of solvable Lie groups and discuss some solutions.

Samir Kabbaj: Equations Fonctionnelles Classiques.

Dans ce travail, on s'intéresse à la résolution de l'équation fonctionnelle

$$\int_G \int_K f(xtk.y)d\mu(t)dk = f(x)f(y) \quad \forall x,t,y \in G \text{ et } \forall k \in K$$

où G est un groupe localement compact, K un groupe compact opérant continûment sur G muni de sa mesure de Haar normalisée dk, μ est une mesure bornée sur G et f est l'inconnue à déterminée, une fonction continue sur G à valeurs dans $\mathbb{C}$. On montre d'abord que les solutions de l'équation proposée, dans le cas d'une mesure μ idempotente, coincident avec les caractères d'une sous algèbre de $L^1(G)$. Comme résultat principal, on montre que les solutions de l'équation ci-dessus, dans le cas d'une mesure $K-invariante$, donnent naissance à des coefficients de sous représentations irréductibles de la $K-$ *régulière* gauche de G.

Chifune Kai: The linearity of order isomorphisms on regular convex cones.

When we have a regular closed convex cone C in a (finite-dimensional) vector space V, the associated partial order is introduced in V. A bijection on a subset of V (not necessarily continuous) is called an order isomorphism if f and f^{-1} preserves the partial order. We show that if C has no half-line factor, every order isomorphism on the interior of C is a restriction of a linear map on V.

Khalid Koufany: Hua operators, Poisson transform and relative discrete series on line bundle over bounded symmetric domains.

Let $\Omega = G/K$ be a bounded symmetric domain and $S = K/L$ its Shilov boundary. We consider the action of G on sections of a homogeneous line bundle over Ω and the corresponding eigenspaces of G-invariant differential operators. The Poisson transform maps hyperfunction-valued sections of a line bundle over S to the eigenspaces. We characterize the image in terms of twisted Hua operators. For some singular parameters the Poisson transform is of Szegö type whose image is in a relative discrete series; we compute the corresponding elements in the discrete series. This is a joint work with Genkai Zhang.

Sami Kouki: Duflo conjecture for solvable Lie groups.

Let G be an exponential solvable Lie group, $\mathfrak{g}$ its Lie algebra and π a unitary irreducible representation of G wich is square intégrable modulo the center, associated by the Kirillov-Bernat map to a G-orbit Ω. Let H be a closed connected subgroup of G with Lie algebra $\mathfrak{h}$ and $p : \mathfrak{g}^* \to \mathfrak{h}^*$ the restriction map. We say that the representation π is *H-admissible* if its restriction to the subgroup H splits in irreducible representations with finite multiplicities. We shall prove the following conjecture due to Duflo:
The representation π is H-admissible, if and only if, the restriction of p to Ω is proper

on the range $p(\Omega)$. In the case in hand, these two conditions are equivalent to $\mathfrak{g} = \mathfrak{h} + \mathfrak{z}$, where $\mathfrak{z}$ is the center of $\mathfrak{g}$.

Jean Ludwig: The C^*-algebra of a thread-like Lie group.

We describe via Fourrier transform the C^* algebra of a thread-like Lie group as an algebra of operator fields defined over its dual space.

Nobuaki Obata: Quantum Probabilistic Spectral Analysis of Large Graphs.

In recent years, quantum probability has been applied to spectral analysis of graphs, in particular, to the study of spectral properties of the adjacency matrices of large, growing, or random graphs. The basic techniques of quantum probability consists of (i) quantum decomposition; (ii) various concepts of independence and quantum central limit theorems; (iii) partition statistics and moment-cumulant formulae. In this talk we review how the above basic tools are applied to the spectral analysis of graphs. In particular, we focus on relation between product structures of graphs (Cartesian, comb, star, free, etc) and various concepts of independence. We also report some recent achievements on the Manhattan product of digraphs for future direction.

[1] A. Hora and N. Obata: Quantum Probability and Spectral Analysis of Graphs Springer, 2007.

[2] N. Obata: Lecture Notes (Tokyo, Sendai, Wroclaw, Cheongju, etc) available at http://www.math.is.tohoku.ac.jp/ obata/

Sanjay Parui: Generalized Hermite expansions of functions arising from Hardy conditions.

Consider a function f on $\mathbb{R}^N$ which is $O(N)$ finite. Assume that both f and its Dunkl Fourier transform $\mathcal{D}_k(f)$ is bounded by $e^{-\frac{1}{2}a|x|^2}$ for some $0 < a < 1$. We will show that the Generalized-Hermite coefficients of f will have exponential decay.

Khemais Maktouf: Sur la restriction de la représentation métaplectique aux sous-groupes compacts.

In order to study restriction of discrete series to closed subgroups of solvable groups over a p-adic field, it is necessary to compute the restriction of the metaplectic representation to anistropic torus of the metaplectic group. For this, we first give the restriction

of the metaplectic representation to the maximal compact subgroups. We give an explicit description of the occuring irreducible representations or characters. This is a joint work with Pierre Torasso.

Dominique Manchon: Pre-Lie and post-Lie algebras : some applications.

Since the pioneering work of A. Cayley in the middle of the Nineteenth century, vector fields on the affine space are known to be closely related with rooted trees. The modern interpretation of this phenomenon relies on the notion of pre-Lie algebra. The structure of the pre-Lie operad also enters into the picture in an essential way. Replacing the affine space by a more general smooth manifold together with a transitive Lie group action, the correct corresponding algebraic framework is given by post-Lie algebras. Rooted trees must then be replaced by (formal Lie brackets of) planar rooted trees. We will explain this correspondence and describe the structure of the post-Lie operad (joint work in progress with K. Ebrahimi-Fard, A. Lundervold, H. Munthe-Kaas and J-E. Vatne).

Ratnakumar P.K.: Non linear Schrodinger equation and the special hermite operator.

We discuss a well posedness result for the nonlinear Schrödinger associated to the special Hermite operator on $\mathbb{C}^n$. We consider a class of nonlinearities, which includes power type non lineariies, in particular. The results relies of the Strichartz estimate for the Schroedinger propagator for the special Hermite operator. This is a joint work with Vijay Sohani.

Atsumu Sasaki: An application of the classification of visible linear actions to nilpotent orbits.

The notion of (strongly) visible actions on complex manifolds has been introduced by T. Kobayashi to give an unified explanation of various multiplicity-free theorems. Recently, we give a classification of strongly visible actions on complex linear spaces. In this talk, we treat the case where a complex manifold is a nilpotent orbit in a complex Lie algebra. We explain that the action of a compact real form of its adjoint group on a spherical nilpotent orbit is strongly visible. The result on strongly visible actions on complex linear spaces plays an important role to our proof.

Sunduram Thangavelu: Riesz transforms and multipliers for the Grushin operator.

We show that Riesz transforms associated to the Grushin operator $-\Delta - |x|^2\partial_t^2$ are bounded on $L^p(\mathbb{R}^{n+1}), 1 < p < \infty$. We also prove a Hormander-Mihlin type multiplier theoem and study Bochner-Riesz means associated to the Grushin operator.

Khalifa Trimeche: Hypergeometric convolution structure on L^p-spaces and application for the Heckman-Opdam Theory.

We study for the Heckman-Opdam theory, the trigonometric Dunkl intertwining operator and its dual, a hypergeometric convolution structure on L^p-spaces, and the Kunze and Stein phenomenon relative to this theory.

Taro Yoshino: Topological Blow-up.

Suppose that a Lie group G acts on a manifold M. The quotient space $X := G\backslash M$ is locally compact, but not Hausdorff in general. Our aim is to understand such a non-Hausdorff space X. The space X has the crack S. Roughly speaking, S is the causal subset of non-Hausdorffness of X, and especially $X \setminus S$ is Hausdorff.

We introduce the concept of 'topological blow-up' as a 'repair' of the crack. The 'repaired' space $\tilde{X}$ is locally compact and Hausdorff space containing $X \setminus S$ as its open subset. Moreover, the original space X can be recovered from the pair of $(\tilde{X}, S)$.