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### **Abstracts**

#### Victor Abramov: Triple Lie Superalgebras and Applications

Given a matrix Lie algebra one can construct the 3-Lie algebra by means of the trace of a matrix. We show that this approach can be extended to matrix Lie superalgebras if instead of the trace of a matrix we make use of the super trace of a matrix. It is proved that a graded triple commutator of matrices constructed with the help of the graded commutator and the super trace satisfies a graded ternary Filippov-Jacobi identity. In two particular cases we show that the Pauli and Dirac matrices generate the matrix 3-Lie superalgebras, and we find the non-trivial graded triple commutators of these algebras. We propose a Clifford algebra approach to 3-Lie superalgebras induced by Lie superalgebras. We also discuss an application of matrix 3-Lie superalgebras in BRST-formalism.

#### Didier Arnal: Regular abelian linear action

In this talk, we shall consider semi direct product  $G = N \rtimes H$  of two vector groups, which is a simply connected solvable group, not necessarily type 1. Put  $V = N^*$ , if the *H*-orbit Hv of v in V is regular, the Mackey theory associates to this orbit and each character  $\chi$  of H(v), a unitary irreducible representation of G. Now G is type 1 if and only if each orbit Hv is regular, and in this case the Mackey construction describes completely the dual  $\hat{G}$  of G.

Here, we recall a simple characterization of the existence of an invariant Zariski open subset  $\Omega$  in  $N^*$ , such that each orbit in  $\Omega$  is regular. In this case there is an explicit rational mapping  $v \mapsto H(v)$ , regular on  $\Omega$ . Suppose now such an  $\Omega$  exists.

We explicitly describe the set  $\Omega/H$  and compute a natural measure  $d\nu$  on  $\Omega/H$ . Moreover, we prove the existence of a character formula for the orbit  $Hv \subset \Omega$ . This implies that, even if G is not type 1, G has sufficiently good representations: its regular representation is type 1 and a Plancherel theorem holds for G. In fact the measure  $d\nu$  and the character formula give the Plancherel measure of G.



### Salma Azaouzi: The Müntz-Szàsz theorem on some compact extensions of nilpotent Lie groups

Given a strictly increasing sequence of positive integers  $(n_k)_k$ , the MÃijntz-SzÃăsz theorem for completeness of the monomials  $\{x_{n_k}\}_k$  in  $L^2([0,1])$ , and even in  $L^2_c(\mathbb{R})$ , can be extended to the Euclidean motion groups  $G = SO(n) \ltimes \mathbb{R}^n$  and the Euclidean spin groups  $spin(n) \ltimes \mathbb{R}^n$  which are known to be the universal coverings of Euclidean motion groups, in which situation we treat separately the case when n = 2 and  $n \ge 3$  as the structures of these groups differ. Towards such an objective, we rephrase the completeness condition in terms of an integral against multivariable polynomials with exponents depending on the sequence  $(n_k)_k$ . We prove by the way that the Fourier transform of the coordinates functions associated to the Garding vectors of G admit an analytic continuation to the whole complex plane with an exponential domination. These upshots are proved using the Plancherel theory related to the group Fourier transform. In the same context, an analog of this result is shown in the case of the compact extensions of the Heisenberg group.

### Abdelhamid Boussejra: $L^p$ -Poisson integral representations of eigensections of the Hua operator on homogeneous line bundles over hermitian symmetric spaces of tube type

Let G/K be an irreducible hermitian symmetric space of tube type and let  $E_{\nu} = G\tilde{A} \mathring{U}_K \mathbb{C}$  be the homogeneous line bundle over G/K associated to a one dimensional representation  $\tau_{\nu}$  of K. Let  $\mathcal{H}_{\nu}$  be the generalized Hua operator acting on  $C^{\infty}$ -section of  $E_{\nu}$  ( $\mathcal{H}_{\nu}$  is a homogeneous differential operator from the space of  $C^{\infty}$ -sections of  $E_{\nu}$  to the space of  $C^{\infty}$ -sections of the homogeneous vector bundle over G/Kassociated to  $\tau_{\nu} \otimes Ad_K \setminus_{\mathfrak{f}_c}$ ). We give a characterization of the eigensections of  $\mathbf{H}_{\nu}$  that have  $L^p$ -integral representations over the Shilov boundary  $G/P_{max}$  of G/K, via the Poisson transfrom on the homogeneous line bundle  $L_{\xi} = G \times P_{max}\mathbb{C}$  over  $G/P_{max}$  associated to an appropriate one dimensional representation of the standard maximal parabolic subgroup  $P_{max}$ . The obtained characterization extends previous results in the trivial case, see [1,2,3].

- 1. A. Boussejra,  $L^2$ -Poisson integral representations of solutions of the Hua system on the bounded symmetric domain  $SU(n,n)/S(U(n) \times U(n))$ , J. Funct. Anal., 202 (2003), 25-43.
- 2. Boussejra, A.,  $L^p$ -Poisson integral representation of solutions of the Hua system on Hermitian symmetric spaces of tube type, J. Funct. Anal., 235 (2006), 413-429.
- 3. Boussejra, A., Koufany, K., Characterization of Poisson integrals for non-tube bounded symmetric spaces, J. Math. Pures. Appl.,87 (2007), 438-451.

### Mashhour Bani-Ata: On a new cipher system over Galois fields of order 27

The aim of this article is to construct a new cipher system for English alphabetic over finite fields of order 27. This paper is divided into three main sections. In section 1, we give an in-troduction. In section 2 we



give a construction of certain finite fields using Kronecher's theorem. In the last section we construct the transformation key of the cipher and explain how the encryption and the decryption sys-tems work.

### Souhail Bejar: On Calabi-Markus phenomenon and the existence of compact Clifford-Klein forms for some compact extensions of nilpotent Lie groups

Let G be a Lie group, H a closed subgroup of G and  $\Gamma$  a discontinuous group for the homogeneous space G/H. We deal in this talk with the problem of Calabi-Markus phenomenon and the existence of Clifford-Klein form, when G stands for a compact extension of a nilpotent Lie group. This includes the context of Euclidean motion groups  $O_n \ltimes \mathbb{R}^n$ , arbitrary compact extensions of  $\mathbb{R}^n K \ltimes \mathbb{R}^n$ , and Heisenberg motion groups  $U_n \ltimes \mathbb{R}^n$ .

### Sabria Benayed: Müntz-Szàsz analogues for compact extensions Heisenberg groups

Given a Müntz-Szàsz sequence of positive integers  $(n_k)_k$ , the Müntz-Szàsz theorem can be extended to compact extensions of Heisenberg groups. Two analogues will be provided.

#### Abdellatif Bentaleb: Ornstein-Uhlenbeck spectral approximations

The aim goal of this note is to study the heat Ornstein-Uhlenbeck semigroup generated by the operator  $Lf(x) := \Delta f(x) - x \cdot \nabla f(x)$ , acting on the Hilbert space  $L^2(\mathbb{R}^d, \gamma_d)$  with weight the canonical Gaussian measure  $\gamma_d(dx) = (\sqrt{2\pi})^{-d} \exp(-|x|^2/2) dx$ . We use some basic properties of the semigroups  $\left\{ \exp(t \prod_{k=0}^n (L-k) \right\}_{t\geq 0}$ ,  $n \geq 0$ , to analyze a large family of geometric inequalities that does not exist in

the literature and with which reinforced the (integral) Poincaré inequality.

A number of auxiliary results of independent interest are obtained including some approximation properties of weighted  $\mathbb{L}^2(\mathbb{R}^d, \gamma_d)$ -orthogonal projectors onto the space of polynomials of polynomials with degree less m.

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#### Hidenori Fujiwara: Polynomial conjecture for restrictions

Let  $G = \exp \mathfrak{g}$  be a connected and simply connected nilpotent Lie group with Lie algebra  $\mathfrak{g}$ ,  $K = \exp \mathfrak{k}$ a closed connected subgroup of G and  $\pi$  an irreducible unitary representation of G. Let  $U(\mathfrak{g})$  be the enveloping algebra of  $\mathfrak{g}_{\mathbb{C}}$  and put

$$U_{\pi}(\mathfrak{g})^{\mathfrak{k}} = \{ X \in \mathfrak{g}; [\mathfrak{k}, X] \subset ker\pi \}.$$

Our object  $D_{\pi}(G)^{K}$  is the image of  $U_{\pi}(\mathfrak{g})^{\mathsf{T}}$  by  $\pi$ . This algebra is commutative if and only if the restriction  $\pi|_{K}$  of  $\pi$  to K has finite multiplicities.

We assume this situation and study the polynomial conjecture, that is to say, the algebra  $D_{\pi}(G)^{K}$  is isomorphic to the algebra  $\mathbb{C}[\Omega(\pi)]^{K}$  of the K-invariant polynomial functions on the coadjoint orbit  $\Omega(\pi)$  of G corresponding to  $\pi$ .

### Ramzi Fendri: On Baklouti's conjecture for some compact extensions of nilpotent Lie groups

Let G be a Lie group, H a closed subgroup of G and  $\Gamma$  a discontinuous group for the homogeneous space G/H. When G is nilpotent and connected, A. Baklouti conjectured that the parameter space  $\mathscr{R}(\Gamma, G, H)$  admits a open orbit if and only if  $\Gamma$  is finite. We record some known results about the conjecture and study the situation where  $\Gamma$  is finite for general Lie groups. We also study the context of arbitrary compact extensions of  $\mathbb{R}^n$ .

#### Saida Farhani: Théorème d'Oschikiri sur les supervariétés riemannienne

Le but du travail est de donner un résultat sur les superfeuillet ages de codimension 1|0 qui est essentiellement basé sur la (super) formule de Green.

Dans cet exposé on va montrer l'analogue de la formule de Green sur une supervariété compacte. La preuve se fait en deux étapes :

- Soit M une variété différentielle compacte, A = C<sup>∞</sup>(M)[, θ<sub>1</sub>, θ<sub>2</sub>] un faisceau des (super) fonctions sur M, M un A-module localement libre de dimension m|2 et Φ une (super)-forme bilinéaire symétrique non dégénérée à valeurs dans A. Comme dans le cas classique, à partir d'une base quelconque de M on construit une base (super) orthonormée relativement à Φ. On donne l'expression de l'adjoint d'un endomorphisme relativement à Φ. On définit cette partie pour montrer que la trace d'un endomorphisme ne depend pas de la base (super) orthonormée choisie.
- 2) Maintenant on considére une supervarité riemannienne  $(\mathcal{M};g)$ , où  $\mathcal{M} = (M; \mathcal{O}_{\mathcal{M}})$  est une supervarieté avec M est une variété diférentielle compacte

et  $\mathcal{O}_{\mathcal{M}} = \mathbb{C}^{\infty}(M)[, \theta_1, \theta_2]$ . Le faisceau tangent  $\mathcal{T}_{\mathcal{M}}$  à  $\mathcal{M}$  est un  $\mathcal{O}_{\mathcal{M}}$ - module. On a sur  $\mathcal{M}$  une connexion dite de Levi-civita associée à g. Pour tout  $X \in \mathcal{T}_{\mathcal{M}}$  on a l'endomorphisme  $A_X$  de  $\mathcal{T}_{\mathcal{M}}$  défini par  $A_X(Y) = (-1)^{|X||Y|} \bigtriangledown_Y X$ . On montre que sur une (super) carte  $\mathcal{U} = (U; \mathcal{O}_{\mathcal{M}})(U)$ , on a l'expression

$$Ric(X) + stA_X^2 + \bigtriangledown_X divX = div \bigtriangledown_X X$$



D'après la partie 1 cette expression est globale. On construit aussi une m|2-forme globale  $\Omega$  sur  $\mathcal{M}$  dite forme volume. Le (super)-théorème de divergence permet d'écrire

$$\int_{\mathcal{M}} Ric(X) + strA_X^2 + \bigtriangledown_X divX = 0$$

# Zeinab Ghardello: Sublinear elliptic problem under radialty on harmonic NA groups and euclidean spaces

Let Ł be the Laplace operator on  $\mathbb{R}^d$ ,  $d \ge 3$  or the Laplace Beltrami operator on the harmonic NA group (in particular on a rank one noncompact symmetric space). For the equation

$$\mathbf{L}u - \varphi(\cdot, u) = 0$$

we give necessary and sufficient conditions for the existence of entire bounded or large solutions under the hypothesis of radiality of  $\varphi$  with respect to the first variable.

#### Hideyuki Ishi: Siegel integral formula for a wide class of convex cones

The cone  $\mathcal{P}_n$  of positive definite real symmetric matrices of size n is a natural multi-dimensional generalization of the half line  $\mathbb{R}^+ = (0, +\infty)$ , and we have various analytic formulas over  $\mathcal{P}_n$  as analogues of ones over  $\mathbb{R}^+$ . Among them, the so-called Siegel integral formula involving the multi-variate Gamma function is particularly important in various areas of pure and applied mathematics. A similar formula for the Lorentz cone is considered by M. Riesz in his study of the wave equation, and the two formulas are unified by Gindikin in the general theory of homogeneous cones. On the other hand, Siegel-type formulas are studied by statisticians for subcones of  $\mathcal{P}_n$  consisting of symmetric matrices with prescribed zero-components (Graphical model).

In this talk, we introduce a class of convex cones realized as specific subcones of  $\mathcal{P}_n$ . The class includes both homogeneous cones and the statistical cones mentioned above. Then we present a Siegel integral formula for this class of cones, and discuss some applications.

# Junko Inoue: Some examples of holomorphically induced representations of solvable Lie groups

We study holomorphically induced representations of exponential solvable Lie groups G. Let f be a real linear form of the Lie algebra  $\mathfrak{g}$  of G, which is extended to  $\mathfrak{g}_{\mathbb{C}}$  complex linearly, and  $\mathfrak{h}$  be a complex subalgebra of  $\mathfrak{g}_{\mathbb{C}}$  satisfying  $f([\mathfrak{h},\mathfrak{h}]) = \{0\}$ . We define a holomorphically induced representation  $\rho$  of G associated with f and  $\mathfrak{h}$ . We investigate some new examples where those subalgebras  $\mathfrak{h}$  are not necessarily polarizations at f, and discuss conditions for non-triviality of  $\rho$  and those decompositions into direct integrals of irreducible representations of G.



#### Samir Kabbaj: On the frames in Hilbert A-modules

We introduce and study the following concepts :

- i) the operator frame for  $End^*_A(H)$  where A is a  $C^*$ -algebra and H is an Hilbert A-module,
- ii) the K operator frame,
- iii) the tensor product of two \* g frames,
- iv) the stability of operator and K operator frame under a small perturbations.

For the first and the second concept we generalize some already known results on the Hilbert space provided with usual scalar product.

For the third concept, we introduce the definition of an \* - g - Frame and we study the tensor product of two of them.

for the fouth concept we study the Ulam's stability of discrete and continous operator frame and K-operator frame.

# Khalid Koufany: Conformally covariant bi-differential operators on a simple real Jordan algebra

For a simple real Jordan algebra V, a family of bi-differential operators from  $\mathcal{C}^{\infty}(V \times V)$  to  $\mathcal{C}^{\infty}(V)$  is constructed. These operators are covariant under the rational action of the conformal group of V. They generalize the classical *Rankin-Cohen* brackets (case  $V = \mathbb{R}$ ). Our construction is based on Bernstein type identities and delicate Fourier calculations using the local Zeta functional equations for V. This is a joint work with Salem Bensaid and Jean-Louis Clerc.

### Edi Kurniadi: Harmonic analysis on 4-dimensional real exponential solvable Frobenius Lie algebras

Based on Csikós-Verhóczki's classification of 4-dimensional Frobenius Lie algebras, we present results on unitary irreducible representations of their Lie groups G obtained by the orbit method, especially in the case when G is an exponential solvable Lie group. Furthermore when G is a semidirect product  $N \rtimes H$ of a unimodular Lie group N and a subgroup H of Aut(N), following Ishi's formulas of continuous wavelet transforms, we consider a realization of its unitary irreducible representation as a square integrable subrepresentation of G on  $L^2(N)$  corresponding to an open orbit.

#### Abdenacer Makhlouf: Rota-Baxter operators

Rota-Baxter operators appeared first in the realm of probability by G. Baxter and then developed from the algebraic viewpoint in combinatorics by G.-C. Rota. A connection to mathematical physics was given



by A. Connes and D. Kreimer in their Hopf algebra approach to renormalization in Quantum field theory. Since then they were intensively studied by providing connections with Yang-Baxter equation and various nonassociative algebras, like dendriform algebras. Recently, some generalizations were studied. Bai, Guo and Ni introduced the extended O-operator generalizing the concept of O-operators and studied the relations with the associative Yang-Baxter equations. While T. Brzezinski introduced the notion of Rota-Baxter system, their curved version and investigated the relations with weak pseudotwistors, differential graded algebras and pre-Lie algebras. The aim of this talk is to review the recent developments and introduce the notion of curved O-operator systems and Rota-Baxter bialgebras. Moreover, we introduce the notion of curved O-operator systems and their connection to some algebraic structures. This is joint work with S. Silvestrov and T. Ma.

#### Salah Mehdi: Asymptotics of characters

Admissible representations are building blocks of representation theory of real reductive Lie groups. Moreover, Harish-Chandra proved that an irreducible admissible representation of such a group G is completely determined by the  $(\mathfrak{g}, K)$ -module of its K-finite vectors and its global character. The global character is an invariant distribution which can be represented on each Cartan subgroup by a conjugacy-invariant function on a dense subset of the semisimple elements. On the other hand, any irreducible  $(\mathfrak{g}, K)$ -module with regular infinitesimal character belongs to a unique coherent family  $\{X_\lambda\}$ . It is therefore important to understand how the restriction, to a Cartan subgroup, of the global character varies in terms of the infinitesimal characters  $\lambda$ . Under the assumption that G has a non empty discrete series, we will explain how Dirac cohomology defines a polynomial on the dual of the compact Cartan. This polynomial expresses the dependence on  $\lambda$  of the leading term in the Taylor expansion of the global character of  $X_\lambda$  on the compact Cartan. Moreover, in the case of representations of Gelfand-Kirillov dimension at most half the dimension of G/K, we prove an explicit relationship between our *Dirac polynomial* and the multiplicities of the irreducible components occuring in the associated cycle of the coherent family  $\{X_\lambda\}$ . This is joint work with Pavle Pandžić (Zagreb), David Vogan (Cambridge) and Roger Zierau (Stillwater).

# Dominique Manchon: About Lie groupoids and their unitary representations

Lie groupoids, in some sense, interpolate between smooth manifolds and Lie groups. We will give an account of the Poisson structure on the dual of the Lie algebroid of a Lie groupoid G, formulate a Kirillov-type conjecture relating the space of symplectic leaves with the unitary representations of  $C^*(G)$ , and illustrate it on various examples.

### Lobna Merghni: Contact points and Schatten class of composition operators

We study the composition operators on the Dirichlet spaces belonging to Schatten class and the link with the size of contact points of its symbol with the unit circle.



# Mostafa Maslouhi: Probabilistic tight frames and representation of positive operators valued matrices

We solve an open problem related to the representation of Positive Operator Valued Matrices by means of tight probabilistic frames in  $\mathbb{R}_d$ . Also, we investigate how far is the closest probabilistic tight frame from a given probability measure where the distance used is the quadratic Wasserstein metric  $W_2$  for measures.

#### Selma Negzaoui Bejaoui: Lipschitz conditions in Laguerre hypergroup

In this talk we show analogous of Titchmarsh's theorems for the Laguerre transform. More precisely, we give a Lipschitz type condition on f in  $L^p(\mathbb{K})$  for which its Laguerre transform belongs to  $L^{\beta}(\hat{\mathbb{K}})$  for some values of  $\beta$ , where  $\mathbb{K} = [0, +\infty) \times \mathbb{R}$  and  $\hat{\mathbb{K}}$  is its dual. In the particular case, when p = 2, we provide equivalence theorem : we get a characterization of the space  $Lip_{\alpha}(\gamma, 2)$  of Lipschitz class functions by means of asymptotic estimate growth of the norm of their Laguerre transform for  $0 < \gamma < 1$ . Furthermore, we introduce Laguerre-Dini-Lipschitz class  $LDLip_{\alpha}(\gamma, \delta, p)$  and we obtain analogous of Titchmarsh's theorems in this occurrence.

### Shimeno Nobukazu: Spherical functions for small *K*-types

For a connected simple real Lie group G of non-compact type, Wallach introduced a class of K-types called small. We classify all small K-types for all simple Lie groups and prove except just one case that the spherical function for any small K-type can be expressed as a product of hyperbolic cosines and a Heckman-Opdam hypergeometric function. As an application, the inversion formula for the spherical transform on the homogeneous vector bundle associated with a small K-type is derived from Opdam's theory of the hypergeometric Fourier transform. This talk is based on joint work with Hiroshi Oda.

### Atsumu Sasaki: A Cartan decomposition for spherical homogeneous spaces of reductive type

Cartan decomposition for semisimple symmetric spaces is an important structural result. In fact, it plays an crucial role to study harmonic analysis on them. On the other hand, there is no analogue of Cartan decomposition for non-symmetric homogeneous spaces. In this talk, we consider a class of homogeneous spaces of reductive type, namely, spherical homogeneous spaces which are in connection with multiplictyfree representations. We explain a generalization of Cartan decomposition to spherical homogeneous spaces of reductive type.



### Mohamed Selmi: Moment and Gelfand transform for enveloping algebra

Describing the Gelfand construction for analytic states on an universal enveloping algebra, we characterize pure states and re-find the main result of a preceding work with L. Abdelmoula and J. Ludwig on the separation of unitary irreducible representations of a connected Lie group by their generalized moment sets.

### Yuichiro Tanaka: Visible actions of compact Lie groups on complex spherical varieties

With the aim of uniform treatment of multiplicity-free representations of Lie groups, T. Kobayashi introduced the theory of visible actions on complex manifolds. In this talk we consider visible actions of a compact real form of a connected complex reductive algebraic group G on spherical varieties. Here a connected complex G-variety X is said to be spherical if a Borel subgroup of G has an open orbit on X. The sphericity implies the multiplicity-freeness property of the space of regular functions on X. We reduce the visibility for the general case to the case of smooth affine spherical varieties by using the method of induction of visible actions. The method of induction was introduced by Kobayashi (2005) for the case of complex spherical nilpotent orbits of type A, and extended by A. Sasaki (2016) to the case of arbitrary type. Our proof is highly motivated by those earlier results.

### Yoshinori Tanimura: An application of F-method to nilpotent homogeneous spaces

F-method was introduced by Toshiyuki Kobayashi to consider the symmetric breaking operator and is a useful tool for dealing with differential operators on symmetric spaces or homogeneous spaces. In the previous research, this method has been used in the case of reductive type homogeneous spaces. In this talk, we will apply this method in the case of nilpotent homogeneous spaces.

#### Tilmann Wurzbacher: Multisymplectic surprises

After briefly recalling how multisymplectic structures (i.e. closed non-degenerate differential forms of degree higher than two) arise in classical field theory, we will give some natural examples of these structures. We will then stress differences to the symplectic case by studying, e.g., the linear and local equivalence problem, and the nature of multisymplectic symmetries.