# Geometric and Harmonic Analysis on Homogeneous Spaces

## Kerkennah, November 2-7, 2009

# Abstracts

## Ahmed ABOUELAZ

**The explicit inversion formulas for the Radon transform in the torus $\mathbb{T}^n$ and its dual $\mathbb{Z}^n$**

**Abstract**. We establish the explicit inversion formulas for the Radon transform in the torus $\mathbb{T}^n$ and also on the lattice $\mathbb{Z}^n$, by using certain arithmetical geometrical techniques.

## Boujemaa AGREBAOUI

**On cohomology and deformations of the Lie superalgebra of contact vector fields on $S^{1|m}$**

**Abstract**. The classical deformation theory of Lie algebras and modules over Lie algebras traditionally deals with one parameter deformations (see M. Gerstanhaber, A. Nijenhuis and R. W. Richardson.) It is, however, natural to consider, as in other deformation theories, multi-parameter deformations. This viewpoint was initiated by A. Fialowski and D. Fuks. The Constructions of deformations of the natural embedding of $\mathrm{Vect}(S^1)$ of vector fields in the circle $S^1$ inside the Poisson algebra of Laurent series on $\dot{T}^*(S^1)$ (resp. the Lie algebra of pseudodifferential operators ) are considered by V. Ovsienko and C. Roger . The multiparameter deformations of the Lie derivative action of the Lie algebra $\mathrm{Vect}(\mathbb{R}^n)$ of vector fields on $\mathbb{R}^n$ on the space of symmetric andantisymmetric tensor fields was considered by B.Agrebaoui F.Ammar, M.Ben ammar,N.Ben Fraj, V.Ovsienko and P. Lecomte.

The first step of any approach to the deformation theory consists in the study of infinitesimal deformations. Given a Lie algebra (or superalgebra) $\mathfrak{g}$ and a $\mathfrak{g}$-module $V$, the infinitesimal deformation is defined, up to equivalence,by the cohomology classes $c_1, \dots, c_n$ in $H^1(\mathfrak{g}, \mathrm{End}(V))$.

The second step is to compute the integrability conditions of infinitesimal deformations.

For the vector fields Lie superalgebra case, the first examination apperies in the end of the seminal paper on modular forms by P.B. Cohen, Y. Manin, and D. Zagier. In my talk, I'll speak about a series of recent papers with N.Ben Fraj, S.Omri and S. Mansour, where the pseudodifferential operator module-valued first cohomology groups

cohomology of the Lie superalgebra $\mathcal{K}(n)$ of contact vector fields on $S^{1|n}$ (or $\mathbb{R}^{1|n}$) is considered and multiparameter deformation has been studied.

The main tools of this lecture are in the papers:

- On the Cohomology of the Lie Superalgebra of contact vector fields on $S^(1|2)$. Non-linear Mathematical Physics Vol 13 (2006) Number 4 523-534 (with N.Ben Fraj et Salem Omri).

- On the cohomology of Lie superalgebra of contact vector fields on $S^{1|m}$ Accepté pour publication au journal Communications in Algebra (with Sabeur Mansour).

-Deforming the Lie Superalgebra of Contact Vector Fields on $S1|2$. Accepted for publication in journal of Geometry and Physics (with Sabeur Mansour).

## Didier ARNAL

### Diamond cones and quasistandard Young tableaux

**Abstract**. In this lecture, we shall recall the notion of shape algebra $S$ for a semisimple Lie algebra $\mathfrak{g}$. Then we define the diamond cone $C$ of $\mathfrak{g}$ as a quotient of $S$.

Diamond cone was introduced by N.J. Wildberger for $\mathfrak{sl}(3)$. If $\mathfrak{n}$ is the nilpotent part of the Iwasawa decomposition of $\mathfrak{g}$, the diamond cone is a $\mathfrak{n}$ module, built from the collection of all maximal, locally nilpotent $\mathfrak{n}$ modules.

To understand the structure of the $\mathfrak{n}$ module $C$, we look for an explicit, combinatorial basis for $C$. Generally speaking, such a basis is given by particular Young tableaux: the quasistandard Young tableaux. We shall present here explicit results for the following cases:

$\mathfrak{g} = \mathfrak{sl}(m)$ (with N. Bel Baraka and N.J. Wildberger),

$\mathfrak{g} = \mathfrak{sp}(2m)$ (with O. Khlifi),

$\text{rank}(\mathfrak{g}) = 2$ (with B. Agrebaoui and O. Khlifi),

$\mathfrak{g} = \mathfrak{sl}(m, 1)$ (this generalization is due to O. Khlifi).

## Najib BEN SALEM

### Some applications of harmonic analysis associated with the Jacobi-Dunkl operators

**Abstract**. The Jacobi-Dunkl operators are differential-difference operators of the form:

$$\Lambda_{\alpha,\beta}(f)(x) = \frac{d}{dx}f(x) + \frac{A'_{\alpha,\beta}}{A_{\alpha,\beta}(x)}(\frac{f(x) - f(-x)}{2})$$

where

$$A_{\alpha,\beta}(x) = 2^{2p}(\sin h^2(x))^{\alpha+\frac{1}{2}}(\cos h^2(x))^{\beta+\frac{1}{2}}, x \in \mathbb{R}, \text{ in the non compact case}$$

and

$$A_{\alpha,\beta}(x) = 2^{2p}(\sin^2(x))^{\alpha+\frac{1}{2}}(\cos^2(x))^{\beta+\frac{1}{2}}, x \in [-\frac{\pi}{2}, \frac{\pi}{2}], \text{ in the compact case.}$$

Here $\alpha \geq \beta - \frac{1}{2}, \alpha \neq -\frac{1}{2}$ and $\rho = \alpha + \beta + 1$.

In the first case, the differential-difference equation

$$\begin{cases} \bigwedge_{\alpha,\beta} \upsilon(x) & = -\lambda\upsilon(x), \lambda \in \mathbb{C} \\ \upsilon(0), & = 1, \end{cases}$$

admits a unique $C^\infty$ solution $\psi_\lambda^{\alpha,\beta}$ on $\mathbb{R}$ given by

$$\begin{cases} \psi_\lambda^{\alpha,\beta}(x) & = \varphi_\mu^{\alpha,\beta}(x) + \frac{id}{\lambda dx}\varphi_\mu^{\alpha,\beta}(x) if \lambda \setminus \{0\} \in \mathbb{C} \\ \psi_0^{\alpha,\beta}(x), & = 1, \end{cases}$$

where $\varphi\mu$ is the Jacobi function and $\lambda^2 = \mu^2 + \rho^2$.
While, in the second case, the differential-difference equation

$$\begin{cases} \bigwedge_{\alpha,\beta} \upsilon(\theta) & = -i\lambda_n\upsilon(\theta), n \in \mathbb{Z} \\ \upsilon(0) & = 1, \end{cases}$$

admits a unique $C^\infty$ solution $\psi_n^{\alpha,\beta}$ on $]-\frac{\pi}{2}, \frac{\pi}{2}[$ given by

$$\begin{cases} \psi_n^{\alpha,\beta}(\theta) & = P_{|n|}^{\alpha,\beta}(\cos(2\theta)) + \frac{i}{\lambda_n}\frac{d}{d\theta}P_{|n|}^{\alpha,\beta}(\cos(2\theta)) if \lambda \in \mathbb{Z}\setminus\{0\} \\ \psi_0^{\alpha,\beta}(\theta) & = 1, \end{cases}$$

where $P_{|n|}^{\alpha,\beta}$ is the Jacobi polynomial of degree $|n|$ and order $(\alpha, \beta)$, $\lambda_n = 2sgn(n)\sqrt{|n|(|n| + \rho)}$.
With the help of the properties of Jacobi functions and polynomials and using harmonic analysis associated with these operators, we study some applications, especially, we deal with Poisson and conjugate Poisson integrals, Hilbert transforms, also we prove the boundedness of such operators.

**Radhouane DAHER**

### Further results in Dunkl harmonic analysis

**Abstract**. We obtain new inequalities for Dunkl transform. There are expressed as gauge on the size of the transform in terms of a suitable integral modulus of continuity of the function. We prove also an analogue of Titchmarch's theorem and Jackson's theorem for Dunkl transform. (This a joint work with Prof M. Maslouhi.)

## Sami DHIEB

**When the deformation space $\mathscr{T}(\Gamma, H_{2n+1}, H)$ is a manifold**

**Abstract**. Let $H_{2n+1}$ be the $2n+1$-dimensional Heisenberg group and $H$ a connected Lie subgroup of $G$. Given any discontinuous subgroup for $G/H$, we know a precise layering into open sets of the resulting deformation space $\mathscr{T}(\Gamma, H_{2n+1}, H)$. We study in this talk when this space is endowed with a smooth manifold structure. (Joint work with Ali Baklouti and Khaled Tounsi)

## Michel DUFLO

**Weyl's functional calculus and equivariant differential forms**

**Abstract**. Let $A_1, A_2, ..., A_d$ be d Hermitian matrices of size $n$. Weyl's functional calculus is a compactly supported distribution $W$ on $\mathbb{R}^d$ which associates to a smooth function $f$ of $d$variables a matrix $W(f) := f(A_1, ..., A_d)$. Forty years ago, Edward Nelson gave a formula for $W$, explicitly describing it as the derivative of a probability measure on $\mathbb{R}^d$ supported on the joint numerical range of the $A_i$. We show how this formula fits in the setting of Hamiltonian geometry and equivariant differential forms.

## Ahmed FITOUHI

**An essay toward a unified theory of the $L^p$ version of Hardy's inequality and $q$-analogues**

**Abstract**. In this paper, we give an $L^p$ version of Hardy's uncertainty principles for a large class of integral transforms. As applications, we discuss an $L^p$ version of Hardy's theorem for the generalized Fourier transform associated to the Sturm-Liouville operator and for the Jacobi-Dunkl transform.

## Hidenori FUJIWARA

**Monomial representations with multiplicities of discrete type**

**Abstract**. Let $G = \exp \mathfrak{g}$ be an exponential solvable Lie group with Lie algebra $\mathfrak{g}$ and $\widehat{G}$ the unitary dual of $G$. Let $H = \exp \mathfrak{h}$ be a closed connected subgroup of $G$ and $\chi$ a unitary character of $H$. We construct the induced representation $\tau = \mathrm{ind}_H^G \chi$ of $G$ and consider the canonical irreducible decomposition of $\tau$:

$$\tau \simeq \int_{\widehat{G}}^{\oplus} m(\pi)\pi d\mu(\pi)$$

with a Borel measure $\mu$ on $\widehat{G}$ and the multiplicity function $m(.)$. We discuss some topics concerning $\tau$ when the data $(G, H, \chi)$ satisfy a certain condition. This a joint work with Ali Baklouti and Jean Ludwig.

## Junko INOUE

### Estimate for the norm of the $L^p$-Fourier transform on nilpotent Lie group

**Abstract**. Let $G$ be a connected nilpotent Lie group, $\tilde{G}$ be its universal covering group, $G = \tilde{G}/\Gamma$, where $\Gamma$ is a discrete subgroup, and let $1 < p \leq 2$, $q$ be such that $\frac{1}{q} + \frac{1}{p} = 1$. We discuss the $L^p$-Fourier transform $\mathcal{F}^p(G)$ on $G$ and obtain an estimate for the norm as follows: Let $\Lambda = \log \Gamma$, $\mathfrak{h} = \mathbb{R}\text{-span}(\Lambda)$, $\tilde{H} = \exp \mathfrak{h}$, $H = \tilde{H}/\Gamma$. Then we have $\|\mathcal{F}^p(G)\|_q \leq A_p^\nu$, where $\nu = \frac{1}{2}(2 \dim(G/H) - m)$, $m$ is the dimension of the generic coadjoint orbits, and $A_p = (p^{\frac{1}{p}}/q^{\frac{1}{q}})^{\frac{1}{2}}$. (This is a joint work with Ali Baklouti.)

## Hideyuki ISHI

### Equivariant holomorphic line bundles over a bounded homogeneous domain

**Abstract**. Let $D$ be a bounded homogeneous domain and $G$ a split solvable Lie group acting on $D$ simply transitively. We shall see that $G$-equivariant holomorphic line bundles $L$ over $D$ are parametrized by one-dimensionel representations $\chi$ of $G$ as $L = L_\chi$. We shall also determine all $\chi$ for which the representation $\pi_\chi$ of $G$ defined naturally on the space $\Gamma_{\text{hol}}(L_\chi)$ is unitarizable. The classification of such unitary representation $\pi_\chi$ will be also given.

## Philippe JAMING

### Distributions that are convolvable with Poisson kernels

**Abstract**. In this talk, we will characterise all distributions on a homogeneous Lie groups that can be extended via a convolution with a Poisson (type) kernel. To do so, we will take an appropriate definition of convolution of distributions and show that distributions that are convolvable with Poisson kernels are derivatives of appropriately weighted $L^1$ functions. Moreover, the result of this convolution has all the expected properties (harmonicity, Fatou type theorem...). (This is joint work with E. Damek, J. Dziubanski and S. Prez-Esteva.)

## Chifune KAI

**The representative domain of a homogeneous bounded domain**

**Abstract**. The representative domain introduced by S. Bergman gives a nice realization for a homogeneous bounded domain, which is a generalization of the Harish-Chandra realization for a symmetric bounded domain. We show that the representative domain coincides with the image of the Cayley transform introduced by R. Penney and T. Nomura. As an application, we see that a homogeneous bounded domain is symmetric if and only if its representative domain is convex.

## Takeshi KAWAZOE

**Hardy space for Jacobi analysis and its applications**

**Abstract**. Let $(\mathbb{R}_+, \Delta)$ denote a space equipped with the measure $\Delta(x)dx$, where

$$\Delta(x) = (2\text{sh}x)^{2\alpha+1}(2\text{ch}x)^{2\beta+1},$$

$x \in \mathbb{R}^+$, $\alpha \geq \beta \geq -1/2$, and a suitable convolution associated with Jacobi function $\phi_\lambda(x)$. Since $\Delta(x)$ has an exponential growth order, $(\mathbb{R}_+, \Delta)$ is not of homogeneous type. A real Hardy space $H^1(\Delta)$ on $(\mathbb{R}_+, \Delta)$ is induced by using radial maximal operator as a usual manner. Then, $H^1(\Delta)$ is related with the classical weighted $H^1$ Hardy spaces on $\mathbb{R}$, especially, it can be characterized in terms of weighted Tribel-Lizorkin spaces on $\mathbb{R}$. Hence, we can obtain an atomic decomposition of $H^1(\Delta)$ and the interpolation space between $H^1(\Delta)$ and $L^1(\Delta)$ that coincides with $L^p(\Delta)$. As an application, we shall consider Fourier multiplier for Jacobi transform $\widehat{f}(\lambda) = \int_0^\infty f(x)\phi_\lambda(x)\Delta(x)dx$.

## Imed KEDIM

**On discontinuous subgroups acting on homogeneous spaces**

**Abstract**. In this lecture, we give a short and basic historical introduction to the theory of deformations associated to Clifford-Klein forms. We present an algebraic formula of this spaces in the setting of completely solvable Lie groups. As an application, we describe this space in the case of the Heisenberg group.(Based on joint works with Ali Baklouti).

## Mohamed Salah KHALGUI

**Classe des algèbres de Lie quasi-réductives**

**Abstract**. Nous étudions une classe d'algèbres de Lie, appeles algèbres de Lie quasi-réductives, liée à l'étude des séries discrètes des groupes de Lie réels et ayant des relations avec les espaces préhomogènes au sens de Kimura-Sato.

## Fatma KHLIF

**Deforming discontinuous subgroups for threadlike homogeneous spaces**

**Abstract**. Let $G$ be an exponential solvable Lie group and $H$ a connected Lie subgroup of $G$. Given any discontinuous subgroup $\Gamma$ for the homogeneous space $\mathscr{M} = G/H$ and any deformation of $\Gamma$, the deformed discrete subgroup may utterly destroy its proper discontinuous action on $\mathscr{M}$ as $H$ is not compact (except the case when it is trivial). To understand this specific issue, we provide an explicit description of the parameter and the deformation spaces of any abelian discrete $\Gamma$ acting properly discontinuously and fixed point freely on $G/H$ for an arbitrary $H$ of a threadlike nilpotent Lie group $G$. The topological features of deformations, such as the local rigidity and the stability are also discussed. Whenever the Clifford-Klein form $\Gamma\backslash G/H$ in question is assumed to be compact, these spaces are cutely determined and unlike the case of Heisenberg groups, the deformation space fails in general to be a Hausdorff space. In such a case, this space is shown to admit a smooth manifold as its open dense subset. (This is a joint work with Ali Baklouti).

## Jean LUDWIG

**Simple modules of the $L^1$ group algebra of $SL_2(\mathbb{R})$**

**Abstract**. We determine the simple modules of the algebra $L^1(SL_2(\mathbb{R}))$ up to equivalence and we show that these modules are the finite rank submodules of the $L^p$-principal series and of the discrete series representations of $SL_2(\mathbb{R})$. This is a joint work with A. PASQUALE.

## Khemais MAKTOUF

**The Plancherel formula for solvable $p$-adic groups**

**Abstract**. In this talk, we prove a Plancherl formula for almost connected solvable $p$-adic groups ($p \neq 2$). To do this, we give a necessary and sufficient condition for a unitary irreducible representation to be admissible in terms of Kirillov-Duflo orbit method. Then we establish a character formula around each semi-simple element.

## Dominique MANCHON

**A deformation approach of the Kirillov map for exponential groups**

**Abstract**. We consider a particular set of ideals of the $C^*$-algebra of the tangent groupoid of an exponential Lie group $G$, such that the evaluation $ev_1$ at $t = 1$ maps this set onto

the unitary dual of $G$, whereas the evaluation $ev_0$ at $t = 0$ identifies it with the set of coadjoint orbits. The composition $ev_1 \circ ev_0^{-1}$ is the Kirillov map. We will indicate how an alternative proof of the Leptin-Ludwig bicontinuity theorem should be expected along these lines. Joint work in progress with A. Baklouti and S. Dhieb.

## Sho MATSUMOTO

**Orthogonal matrix integrals and zonal spherical functions name**

**Abstract**. We consider about integrals of polynomial functions of Haar-distributed orthogonal matrix entries. Benoit Collins and Piotr Sniady proved that the integrals are evaluated by using special functions on symmetric groups. The functions are called Weingarten functions. Our main result is to give an expansion of the Weingarten function with respect to zonal spherical functions of a finite Gelfand pair. Moreover, we derive a connection between Weingarten functions and Jucys-Murphy elements. This talk is a part of the joint work with Benoit Collins.

## Carine MOLITOR-BRAUN

**Flat orbits, primary ideals and spectral synthesis**

**Abstract**. Let $G = \exp \mathfrak{g}$ be a connected, simply connected, nilpotent Lie group and let $\omega$ be a continuous symmetric weight on $G$ with polynomial growth. Let $l \in \mathfrak{g}^*$ be such that the co-adjoint orbit of $l$ is flat. In the weighted group algebra $L^1_\omega(G)$ we characterize all the twosided closed ideals whose hull is $\{\pi_l\}$, where $\pi_l$ denotes the element of $\hat{G}$ associated to the co-adjoint orbit $\mathrm{Ad}^*(G)(l)$ by the Kirillov map. These ideals are parametrized by a set of $G$-invariant, translation invariant spaces of complex polynomials dominated by the weight $\omega$ and are realized as kernels of specially built induced representations. This results among others from the fact that, if the co-adjoint orbit of $l$ is flat, every closed two-sided ideal of $L^1_\omega(G)$ with hull $\{\pi_l\}$ is necessarily $L^\infty(G/G(l))$-invariant, where $G(l)$ denotes the stabilizer of $l$ in $G$. (This is a joint work with J. Ludwig).

## Atsumu SASAKI

**Visible actions on complex vector spaces**

**Abstract**. I plan to explain that the multiplicity-free space has a strongly visible actions. Let $V$ be a vector space over $\mathbb{C}$ and $G_\mathbb{C}$ be a connected complex reductive Lie group. Given a holomorphic representation of $G_\mathbb{C}$ on $V$, we define a repesentation $\pi$ on the polynomial ring $\mathbb{C}[V]$ of $G_\mathbb{C}$. We show that the polynomial representation $\pi$ is multiplicity-free if and only if the action of a maximal compact subgroup of $G_\mathbb{C}$ on $V$ is strongly visible.

## Mohamed SELMI

**Séparation des représentations par les sur-groupes quadratiques**

**Abstract**. Let $\pi$ be an unitary irreducible representation of a Lie group $G$. $\pi$ defines a moment set $I_\pi$, subset of the dual $\mathfrak{g}^*$ of the Lie algebra of $G$. Unfortunately, $I_\pi$ does not characterize $\pi$.
However, we sometimes can find an overgroup $G^+$ for $G$, and associate, to $\pi$, a representation $\pi^+$ of $G^+$ in such a manner that $I_{\pi^+}$ characterizes $\pi$, at least for generic representations $\pi$. If this construction is based on polynomial functions with degree at most 2, we say that $G^+$ is a quadratic overgroup for $G$.
In this paper, we prove the existence of such a quadratic over-group for many different classes of $G$.

## Nobukazu SHIMENO

**Matrix-valued commuting differential operators associated with root systems**

**Abstract**. We obtain matrix-valued commuting differential operators whose coeeficients are elliptic functions, by generalizing explicit expressions of matrix-valued commuting differential operators on homogeneous vector bundles on certain rank 2 symmetric spaces.

## Mohamed SIFI

**Three results in Dunkl analysis**

**Abstract**. In this talk, we establish first a geometric Paley–Wiener theorem for the Dunkl transform in the crystallographic case. Next we obtain an optimal bound for the $L^p \to L^p$ norm of Dunkl translations in dimension 1. Finally we describe more precisely the support of the distribution associated to Dunkl translations in higher dimension. (This is a joint work with B. Amri and J. Ph. Anker).

## Kais SMAOUI

**On Beurling's theorem for nilpotent Lie groups**

**Abstract**. In the case of the real line, the Beurling theorem asserts that for any non trivial function $f \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$, the function $f(x)\widehat{f}(y)$ is never integrable on $\mathbb{R}^2$ with respect to the measure $e^{|xy|}dxdy$. We prove an analogue of this theorem for an arbitrary simply connected nilpotent Lie group extending then earlier cases and the classical Cowling-Price theorem proved recently by A. Baklouti and N. Ben Salah.

# Sundaram THANGAVELU

**A Paley-Wiener theorem for eigenfunction expansions**

**Abstract**. We prove a Paley-Wiener theorem for certain eigenfunction expansions which includes Fourier series on compact Lie groups, spherical harmonic expansions, Hermite and Laguerre series. We characterise functions having finite expansions in terms of growth estimates of their holomorphic extensions. We use the holomorphically extended heat kernels to measure the growth.

# Khalifa TRIMECHE

**Harmonic analysis associated with the Cherednik operators and the Heckman-Opdam theory**

**Abstract**. By using the trigonometric Dunkl intertwining operators and their duels introduced by the author, we define and study the hypergeometric translation operators associated with the Cherednik operators $T_1, ..., T_d$ and the Heckman-Opdam theory. Next with the help of these translation operators we define and study the hypergeometric convolution product of functions and distributions associated with the operators $T_1, ..., T_d$ and the Heckman-Opdam theory.

# Yoshinori YAMASAKI

**Zeta regularized products and higher depth determinants of Laplacians**

**Abstract**. In this talk, as analogues of the Milnor gamma functions, we introduce "higher depth determinants" of Laplacians via zeta regularized products. In particular, we explicitly calculate them in the case of compact Riemann surfaces with negative constant curvature and higher dimensional spheres.