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Hardy space for Jacobi analysis and its applications

Abstract. Let $(\mathbb{R}_+, \Delta)$ denote a space equipped with the measure $\Delta(x)dx$, where

$$\Delta(x) = (2\text{sh}x)^{2\alpha+1}(2\text{ch}x)^{2\beta+1},$$

$x \in \mathbb{R}^+$, $\alpha \geq \beta \geq -1/2$, and a suitable convolution associated with Jacobi function $\phi_\lambda(x)$. Since $\Delta(x)$ has an exponential growth order, $(\mathbb{R}_+, \Delta)$ is not of homogeneous type. A real Hardy space $H^1(\Delta)$ on $(\mathbb{R}_+, \Delta)$ is induced by using radial maximal operator as a usual manner. Then, $H^1(\Delta)$ is related with the classical weighted H^1 Hardy spaces on $\mathbb{R}$, especially, it can be characterized in terms of weighted Tribel-Lizorkin spaces on $\mathbb{R}$. Hence, we can obtain an atomic decomposition of $H^1(\Delta)$ and the interpolation space between $H^1(\Delta)$ and $L^1(\Delta)$ that coincides with $L^p(\Delta)$. As an application, we shall consider Fourier multiplier for Jacobi transform $\widehat{f}(\lambda) = \int_0^\infty f(x)\phi_\lambda(x)\Delta(x)dx$.