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Some applications of harmonic analysis associated with the Jacobi-Dunkl operators

Abstract. The Jacobi-Dunkl operators are differential-difference operators of the form:

$$\Lambda_{\alpha,\beta}(f)(x) = \frac{d}{dx}f(x) + \frac{A'_{\alpha,\beta}}{A_{\alpha,\beta}(x)}(\frac{f(x)-f(-x)}{2})$$

where

$$A_{\alpha,\beta}(x) = 2^{2p}(\sin h^2(x))^{\alpha+\frac{1}{2}}(\cos h^2(x))^{\beta+\frac{1}{2}}, x \in \mathbb{R},\text{ in the non compact case}$$

and

$$A_{\alpha,\beta}(x) = 2^{2p}(\sin^2(x))^{\alpha+\frac{1}{2}}(\cos^2(x))^{\beta+\frac{1}{2}}, x \in [-\frac{\pi}{2},\frac{\pi}{2}],\text{ in the compact case.}$$

Here $\alpha \geq \beta - \frac{1}{2}, \alpha \neq -\frac{1}{2}$ and $\rho = \alpha + \beta + 1$.

In the first case, the differential-difference equation

$$\begin{cases} \bigwedge_{\alpha,\beta} \upsilon(x) & = -\lambda\upsilon(x), \lambda \in \mathbb{C} \\ \upsilon(0), & = 1, \end{cases}$$

admits a unique C^∞ solution $\psi_\lambda^{\alpha,\beta}$ on $\mathbb{R}$ given by

$$\begin{cases} \psi_\lambda^{\alpha,\beta}(x) & = \varphi_\mu^{\alpha,\beta}(x) + \frac{id}{\lambda dx}\varphi_\mu^{\alpha,\beta}(x) if \lambda \setminus \{0\} \in \mathbb{C} \\ \psi_0^{\alpha,\beta}(x), & = 1, \end{cases}$$

where $\varphi\mu$ is the Jacobi function and $\lambda^2 = \mu^2 + \rho^2$.
While, in the second case, the differential-difference equation

$$\begin{cases} \bigwedge_{\alpha,\beta} \upsilon(\theta) & = -i\lambda_n\upsilon(\theta), n \in \mathbb{Z} \\ \upsilon(0) & = 1, \end{cases}$$

admits a unique C^∞ solution $\psi_n^{\alpha,\beta}$ on $]-\frac{\pi}{2},\frac{\pi}{2}[$ given by

$$\begin{cases} \psi_n^{\alpha,\beta}(\theta) & = P_{|n|}^{\alpha,\beta}(\cos(2\theta)) + \frac{i}{\lambda_n}\frac{d}{d\theta}P_{|n|}^{\alpha,\beta}(\cos(2\theta)) if \lambda \in \mathbb{Z}\setminus\{0\} \\ \psi_0^{\alpha,\beta}(\theta) & = 1, \end{cases}$$

where $P_{|n|}^{\alpha,\beta}$ is the Jacobi polynomial of degree $|n|$ and order (α,β), $\lambda_n = 2sgn(n)\sqrt{|n|(|n|+\rho)}$.
With the help of the properties of Jacobi functions and polynomials and using harmonic analysis associated with these operators, we study some applications, especially, we deal with Poisson and conjugate Poisson integrals, Hilbert transforms, also we prove the boundedness of such operators.