

A left invariant pseudometric d on $GL(n, \mathbb{R})$ is called norm-like if there is a norm $\|\cdot\|$ on \mathbb{R}^n such that the functions $(t_1, \dots, t_n) \mapsto \|(t_1, \dots, t_n)\|$ and $(t_1, \dots, t_n) \mapsto d(1, \text{diag}(e^{t_1}, \dots, e^{t_n}))$ on \mathbb{R}^n are of bounded difference. Here $\text{diag}(s_1, \dots, s_n)$ denotes the diagonal matrix with diagonal entries (s_1, \dots, s_n) . Examples of norm-like pseudometrics are the pseudometric coming from the operator norm on $GL(n, \mathbb{R})$ given by any norm on \mathbb{R}^n and the pseudometric coming from the symmetric space. A recent result of Margulis and Abels says that every coarsely geodesic pseudometric (e.g. the word metric with respect to a compact set of generators) on $GL(n, \mathbb{R})$ is norm-like. An analogous result holds for reductive groups over local fields. This research was motivated by a question that Siegel asked in 1959 in his Japan lectures on reduction theory.