Geometric and Harmonic Analysis on Homogeneous Spaces and Applications

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Abstracts

Lobna Abdelmoula : The Selberg-Weil-Kobayashi local rigidity Theorem for exponential Lie groups.

A local rigidity Theorem proved by A. Selberg and A. Weil for Riemannian symmetric spaces and generalized by T. Kobayashi for a non-Riemannian homogeneous space G/H, asserts that there are no continuous deformations of a cocompact discontinuous subgroup Γ for G/H in the setting of a linear non-compact semi-simple Lie group G except some few cases : G is not locally isomorphic to $SL_2(\mathbb{R})$ for H compact or G is not locally isomorphic to $SO(n,1)$ or $SU(n,1)$ for $G \times G$ and $H = \Delta_G$. When in large contrast G is assumed to be exponential solvable and $H \subset G$ a maximal subgroup, we prove an analogue of such a Theorem stating that the local rigidity holds on the parameter space $\mathscr{R}(\Gamma, G, H)$ if and only if G is isomorphic to the two-dimensional group of affine transformations of the line $ax + b$. Remarkably, we do also drop the assumption on Γ to be uniform for G/H. This is a joint work with Ali Baklouti and Imed Kedim.

Luigi Accardi : Infinite dimensional Lie algebras and renormalized higher powers of white noise.

The problem of defining, in an operational way, renormalized powers of free quantum fields (white noises in mathematical language) has been a central one in mathematical physics for the past 50 years (see the paper [3] for a survey).

In 1999 a new, Lie–algebraic, approach to this problem was proposed by Accardi, Lu and Volovich and has led, through the contributions of a multiplicity of authors, to the construction of the renormalized square of (quantum) white noise (RSWN) and to its identification with the theory of unitary representations of the current algebra over $\mathbb{R}^d$ of $sl(2,\mathbb{R})$.

The success and the non-triviality of the theory in the quadratic case naturally rose the problem of extending these results to powers higher than 2.

This program has been systematically pursued by Accardi and Boukas in the past 14 years and has led to the conclusion that the role of $sl(2,\mathbb{R})$, for higher powers is an infinite dimensional Lie algebra 'strictly related' the Virasoro–Zamolodzhikov hierarchy of Lie algebras, introduced in the theory of strings for totally different purposes (the relation

is that the two algebras have the same closure in an appropriately defined topology, see the survey [1]).

This research has brought to light an interesting connection between renormalization and central extensions of Lie algebras as well as several new results on central extensions of infinite dimensional Lie algebras (see the survey [2]).

The present talk will be a survey of the main new ideas emerged in this field with emphasis on open problems, whose solution will require the collaboration of experts of quantum probability with experts of Lie algebras.

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Ahmad M. A. Alghamdi : New Characterization of Lie Triple Systems.

This work is a joint project with Dr. Amir Baklouti. We are studying the concept of Lie Triple Systems. A Lie triple system is a vector space with trilinear map which satisfies the condition of antisymmetry, Jacobi identity and derivative identity. We are trying to get a new characterization of such concept by using the so-called Casimir operator as a tool. A Casimir element (operator) is a distinguished element of the centre of the universal enveloping algebra of a Lie algebra. In this talk, I shall discuss and focus in the use of this operator and mention some properties of this distinguished element.

Souhail Bejar : Deforming discontinuous subgroups of Euclidean motion groups.

Let G be a Lie group and H a connected closed subgroup of G. Given any discontinuous subgroup Γ for the homogeneous space $\mathscr{M} = G/H$ and any deformation of Γ, the deformed discrete subgroup may utterly destroy its proper discontinuous action on $\mathscr{M}$ whenever H is not compact. To understand this specific issue in the setting where

$G := O_n(\mathbb{R}) \ltimes \mathbb{R}^n$ stands for the Euclidean motion groups, we provide an explicit description of the parameter and the deformation spaces of any discrete Γ acting properly discontinuously and fixed point freely on G/H for an arbitrary H. Remarkably, it happens that H turns to be compact whenever Γ is infinite, which means that the proper action resists to any deformation of Γ but probably the free action does not. Furthermore the parameter space has a quite complicated topological features imposing a distinction of two types of stability. The situation where Γ is a crystallographic subgroup is extensively studied. This is a joint work with Ali Baklouti.

Agrebaoui Boujemâa : Jeu de taquin and diamond cone for classical Lie (super)algebras.

Each irreducible module of a complex semisimple Lie super algebra $\mathfrak{g}$ is characterized by its highest weight λ and is generated by the action of $\mathfrak{g}$ on the unique (up to constant) highest weight vector v_λ, with weight λ. The sum of all such modules $\mathbb{S}^\lambda$, has a structure of a commutative associative algebra, denoted $\mathbb{S}$ and called the shape algebra of $\mathfrak{g}$. The classical Weyl theory for $\mathfrak{sl}_n(\mathbb{C})$ and its extension to the case of $\mathfrak{sp}(2n)$, and $\mathfrak{so}(2m+1)$ (see [FH]) gives a cobinatorial basis for the shape algebra in terms of semistandard tableaux. This notion of semistandard tableaux was extended to the case of Lie superalgebras $\mathfrak{sl}(m,n)$ and $\mathfrak{spo}(2n,2m+1)$ (see [CW]). For $\mathfrak{sl}(m,n)$, semistandard tableaux give a basis for the sum of all simple tensor modules (see [BR]).

Consider now the nilpotent factor $\mathfrak{n}$ of $\mathfrak{g}$. The reduced shape algebra $\mathbb{S}_{red}$ is the union of all maximal monogenic modules $\mathbb{S}^\lambda|_{\mathfrak{n}}$. It is the quotient of $\mathbb{S}$ by the ideal generated by all $v_\lambda - 1$.

A quasistandard tableau is a semistandard one from which it is impossible to extract any trivial subtableau by using the Schützenberger's jeu de taquin. An explicit combinatorial model for a basis, called the diamond cone, of $\mathbb{S}_{red}$ in terms of quasistandard tableaux is given in the case of $\mathfrak{sl}_n(\mathbb{C})$ and $\mathfrak{sp}(2n)$(see [ABW, AK]).

We extended these notions to the case of $\mathfrak{so}(2m+1)$ (see [AAB1]), $\mathfrak{sl}(m,n)$ (see [AAK2]) and to $\mathfrak{spo}(2n,2m+1)$ (see [AAB2]).

Références

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Didier Arnal[(1)] and Bradely Currey[(2)] : Regularity and cross-section for multiply generated abelian group actions : I and II.

In two lectures, we present recent joint work with Béchir Dali and Vignon Oussa.

Let $\mathfrak{g}$ be the real span of a finite set of commuting endomorphisms of a real vector space V and put $G = \exp \mathfrak{g}$. To each $G-$orbit in V, we attach a subgroup of a torus, with the property that the orbit is regular if and only if the subgroup is closed. We construct an layering of V into $G-$invariant semi-algebraic subsets in which the rotation subgroups are described by explicit $G-$invariant rational functions. We show that one of the following holds : either every orbit in the generic layer is regular, or there is a co-null, $G-$invariant $\mathfrak{G}_\delta$-subset in which every orbit is not regular. We show that in the a.e. regular case, the functions describing the rotation subgroups for the generic orbits are constant. In each layer, if every orbit is regular, we present an explicit construction of a topological section for the orbits in the layer. We discuss applications to the harmonic analysis of $V \rtimes G$, and provide various examples.

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Salma Azaouzi : A generalized analogue of Hardy's uncertainty principle on compact extensions of $\mathbb{R}^n$. (Phd Thesis)

A classical theorem of Hardy proved in 1933 in the case of the real line, states that an integrable function f and its Fourier transform $\hat{f}$ cannot both have arbitrary Gaussian decay unless f is identically zero. More precisely, if both $f(x)e^{\alpha\|x\|^2}$ and $\hat{f}(\xi)e^{\beta\|\xi\|^2}$ are in $L^\infty(\mathbb{R}^n)$ for some $\alpha, \beta > 0$ then the following conclusions hold :

1. $f = 0$ whenever $\alpha\beta > 1/4$.
2. The function f is a constant multiple of $e^{-\alpha\|x\|^2}$ when $\alpha\beta = 1/4$.
3. When $\alpha\beta < 1/4$, there are infinitely many linearly independent functions satisfying both conditions.

There has been much effort to prove Hardy-like theorems for various classes of non-abelian connected Lie groups. Specifically, analogues of Hardy's theorem have been shown for the Euclidean motion groups (Sundari 1998, Sarkar and Thangavelu 2005, Baklouti-Kaniuth 2010).

Let $\lambda > 0$ and $\sigma \in \widehat{SO(n-1)}$, then we have an irreducible unitary representation $\pi_{\lambda,\sigma}$ of the motion group $SO(n) \ltimes \mathbb{R}^n$. Then functions satisfying the conditions $|f(a,x)| \leq \varphi(a)e^{-\alpha\|x\|^2}$ and $\|\pi_{\lambda,\sigma}(f)\|_{HS} \leq \psi(\sigma)e^{-\beta\lambda^2}$, where $\varphi \in L^2(SO(n))$ and $\psi \in l^2(\widehat{SO(n-1)})$, are proved to be trivial on $SO(n) \ltimes \mathbb{R}^n$ in the only case of $\alpha\beta > \frac{1}{4}$. Now if we let K be a compact subgroup of automorphisms of $\mathbb{R}^n$. Can we replace $SO(n)$ by an arbitrary compact subgroup such as K ? Then in the case of $\alpha\beta = \frac{1}{4}$, can we characterize the functions satisfying similar assumptions as above ? Answering to these questions, I obtained a generalization of Hardy's uncertainty principle on the semi-direct product $K \ltimes \mathbb{R}^n$ (that we denote by G), with a complete characterization of the function f in the three cases $\alpha\beta > \frac{1}{4}$, $\alpha\beta = \frac{1}{4}$ and $\alpha\beta < \frac{1}{4}$.

To prove such a result, we have first of all to know explicitly the unitary dual of G. We can ensure that using the Mackey theory. Once the dual is identified and Fourier transform defined, we estimate the Hilbert-Shmidt norm of the Fourier transform of the function. These two aspects are comparatively more complicated than the case of $K = SO(n)$. Moreover, having proved Hardy's theorem on G, I take up other uncertainty principles. Namely, Cowling-Price, Miyachi, $L^p - L^q$ Morgan, Beurling, Bonami-Demange-Jaming and Gelfand-Shilov theorems, which proofs require an explicit knowledge of the unitary dual of G and of the Hilbert-Shmidt norm of the Fourier transform on $\widehat{G}$.

Abdessatar Barhoumi : White Noise Meixner Processes and Related $\star$-Lie Algebras.

By using an appropriate one-mode type interacting Fock spaces, $\Gamma_{M}(\mathscr{H})$, we derive a chaotic decomposition property for the Hilbert space of quadratic integrable functionals with respect to the Meixner white noise measure Λ_{M}. The constructed decomposition is used to define a nuclear triple $\mathscr{F}_{M,\theta}(\mathscr{E}) \subset L^2(\mathscr{E}', \Lambda_{M}) \subset \mathscr{F}^{*}_{M,\theta}(\mathscr{E})$ of test and generalized functions, with θ being a suitable Young function. Moreover, a general characterization theorems are proven for the fundamental nuclear spaces. For the applications, we introduce a new renormalized products for the generators of the renormalized higher powers of white noise $\star$-Lie algebra and the Virasoro-Zamolodchikov-w_{∞} $\star$-Lie algebra. Then we show that these new renormalized products lead to a nuclear realizations of these famous Lie algebras in terms of quantum Meixner white noise operators.

Nizar Ben Fradj : Cohomology of $\mathrm{osp}(n|2)$ Acting on the Modules of Linear Differential Operators on the Weighted Densities on the Supercircle $S^{1|n}$ and classification of these modules.

We compute the frst differential cohomology of the orthosymplectic Lie superalgebra $osp(n|2)$ with coefficients in the superspace of linear differential operators acting on the space of weighted densities on the Supercircle $S^{1|n}$. We give, in contrast to the classical setting, a classification of these modules. for $n = 1$. We also prove that $\mathfrak{D}^{n}_{\lambda,\mu}$ and $\mathfrak{D}^{n}_{\rho,\nu}$ are isomorphic for $\rho = \frac{2-n}{2} - \mu$ and $\nu = \frac{2-n}{2} - \lambda$. $:= \mathrm{Hom}_{\mathrm{diff}}(\mathfrak{F}^{n}_{\lambda}, \mathfrak{F}^{n}_{\mu})$, where $\mathfrak{F}^{n}_{\lambda}$ is the $\mathscr{K}(n)$-module of λ-densities on $\mathbb{R}^{1|n}$. It is filtered : $\cdots \subset \mathfrak{D}^{n,k}_{\lambda,\mu} \subset \mathfrak{D}^{n,k+\frac{1}{2}}_{\lambda,\mu} \cdots$, where $k \in \frac{1}{2}\mathbb{N}$. We prove that, in ontrast to the classical setting, the $\mathscr{K}(n)$-modules $\mathfrak{D}^{n,k}_{\lambda,\mu}$ and $\mathfrak{D}^{n,k}_{\rho,\nu}$ are isomorphic if and only if $\rho = \frac{2-n}{2} - \mu$ and $\nu = \frac{2-n}{2} - \lambda$. This work is the simplest superization of a result by Gargoubi and Ovsienko [Modules of Differential Operators on the Real Line, Functional Analysis and Its Applications, Vol. 35, No. 1, pp. 13–18, 2001.]

Abdellatif Bentaleb : Some sharp interpolation inequalities of the Poincaré inequality.

The main purpose of this short note is establish and analyze study, via a method involving some semigroup techniques, a new family of sharp integral inequalities related to the Jacobi probability measure $\mu(dx) = cx^{\alpha}(1+x)^{\beta}dx; \alpha, \beta > -1$: These estimations are a natural extension and refinement of the spectral gap and reverse spectral gap

inequalities. New integral inequalities for the uniform measure on a compact interval are also derived with the classical Poincaré inequality as particular case.

Aline Bonami : Bergman projection and Bloch classes in bounded symmetric domains of tube type.

Let D be an irreducible bounded symmetric domain of tube type in $\mathbb{C}^n$. The class of Bloch functions is well known in this context as the space of bounded Hankel operators or as the dual of the Bergman space A^1. Contrarily to what happens in the unit ball, Bloch functions do not belong to all Lebesgue spaces $L^p(D)$ for $p < \infty$ in rank r>1. We will give both necessary and sufficient conditions on p for such an embedding. This question is equivalent to local boundedness properties of the Bergman projection in the tube domain over a symmetric cone that is conformally equivalent to D.

This is a joint work with G. Garrigós and C. Nana, to appear in Journ. Geo. Anal. We will also mention recent work of Cyrille Nana in the non tube case.

Radhouane Daher : On a Theorem of Titchmarsh.

In this paper, we prove an analog of Titchmarsh's theorem in certain sitting. More precisely for the circle group, Bessel-Jacobi hypergroups, non compact symmetric spaces, the sphere, ...we obtain an analog of one classical Titchmarsh theorem on description of the image under the Fourier transform of class of functions satisfying the Lipschitz condition. The results are obtained in various collaborationa with Mohamed ELHAMMA.

Kahar El-Hussein : Existence Theorems and Left Ideal of Group Algebra on the Nilpotent Cartan-Lie Group G_5.

Let G_5 be the 5-dimensional nilpotent Cartan-Lie group. Thanks to its finest structure, this group can be shown as a semi-direct product of three real vector groups. The purpose of this paper is to generalize the Fourier transform and to prove the Plancherel theorem. Moreover we prove some existence theorems for the invariant differential opera- tors on G_5. To this end we give a classification of all left ideals of group algebra $L^1(G)$ of G_5.

Saïd Fahlaoui : On extension and refinement of the Poincaré inequality

The aim of this paper is to analyze the heat semigroup $(N_t)_{t>0} = \{e^{t\Delta}\}_{t>0}$ generated by the usual Laplacian operator Δ on $\mathbb{R}^d$ equipped with the $d-$dimensional Lebesgue measure. We obtain and study, via a method involving some semigroup techniques, a large

family of functional inequalities that does not exist in the literature and with the local Poincaré and reverse local Poincaré inequalities as particular cases. As a consequence, we establish in parallel a new functional and integral inequality related to the Ornstein-Uhlenbeck semigroup.

Jacques Faraut : Multivariate Markov-Krein type formula for projections of orbital measures.

The orthogonal group $U = O(n)$ (resp. the unitary group $U(n)$) acts on the space $V = Sym(n, \mathbb{R})$ of $n \times n$ symmetric matrices (resp. $Herm(n, \mathbb{C})$ of Hermitian matrices) by the transformations $X \mapsto uXu^*$ $(u \in U)$. For $A \in V$,

the orbital measure μ_A is defined by :

$$\int_V f(X)\mu_A(dX) = \int_U f(uAu^*)du.$$

We consider the projection μ_A^k of μ_A on the subspace V_k of $k \times k$ matrices. The measure μ_A^1 on $V_1 \simeq \mathbb{R}$ satisfies the following Markov-Krein type formula

$$\int_{\mathbb{R}} \frac{1}{(z-t)^{n\frac{d}{2}}}\mu_A^1(dt) = \prod_{j=1}^{n} \frac{1}{(z-\lambda_j)^{\frac{d}{2}}} \quad (z \in \mathbb{C} \setminus \mathbb{R}),$$

where $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of A, $d = 1$ in case of $Sym(n, \mathbb{R})$, and $d = 2$ in case of $Herm(n, \mathbb{C})$. By using this formula it is possible to determine the density of the measure μ_A^1 (F. Fourati, 2011, Journal of Lie Theory). For $d = 2$ this density is a spline function.

We will present a multivariate analogue of this formula involving the measure μ_A^k for $k \geq 1$.

Ahmad Fitouhi : q-Bessel-Hahn-Exton transform and applications.

We prove the positivity of the q-generalized translation associated with the q-bessel function published recently in constructive approximation and we give some applictions such definite positive functions q-theory.

Hidenori Fujiwara : Some problems for induced and restricted representations of exponential solvable Lie groups.

We discuss in framework of the orbit method some problems for in- duced and restricted representations of exponential solvable Lie groups ; intertwining operators and polynomial conjectures. (Joint works with Jean Ludwig and Ali Baklouti)

Sonia Ghaour : Deforming discontinuous subgroups of reduced Heisenberg groups.

Let $G = \mathbb{H}^r_{2n+1}$ be the $2n+1$-dimensional reduced Heisenberg group and H an arbitrary connected Lie subgroup of G. Given any discontinuous subgroup $\Gamma \subset G$ for G/H, we show that resulting deformation space $\mathscr{T}(\Gamma, G, H)$ of the natural action of Γ on G/H is endowed with a smooth manifold structure and is a disjoint union of open smooth manifolds. Unlike the setting of simply connected Heisenberg groups, we show that the stability property holds and that any discrete subgroup of G is stable in the sense of A. Baklouti, following the notion of stability introduced by Kobayashi-Nasrin. On the other hand, a local (and hence global) rigidity theorem is obtained. That is, the related parameter space $\mathscr{R}(\Gamma, G, H)$ admits a rigid point if and only if Γ is finite. This is a joint work with Ali Baklouti and Fatma Khlif.

Abdelhamid Hassairi : On Cauchy-Stieltjes Kernel families.

We extend the classification of the Cauchy-Stieltjes kernel families with compactly supported generating measures to cover measures with support unbounded on one side. We illustrate the need for such an extension by showing that cubic pseudo-variance functions correspond to free-infinitely divisible laws without the first moment. We then explore properties of Cauchy-Stieltjes families that have no counterpart in exponential families. We relate the variance function of the iterated Cauchy-Stieltjes family to the pseudo-variance function of the initial Cauchy-Stieltjes family. We also investigate when the domain of means can be extended beyond the "natural domain".

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Junko Inoue : Holomorphically induced representations of some exponential solvable Lie groups.

We are concerned with holomorphically induced representations ρ of exponential Lie groups G starting from weak polarizations or general complex subalgebras of $\mathfrak{g}_{\mathbb{C}}$, the complexification of the Lie algebra of G. We shall discuss problems of non-triviality and description of decomposition of ρ into irreducible representations, treating some concrete examples.

Hideyuki Ishi : Generalized highest weight unitary representations of a split solvable Lie group.

We generalize the notion of highest weight unitary repre- sentation to a real Lie group which is not necessarily reductive. Such representations are classified for a connected and simply connected split solvable Lie group, where our argument is based on Fujiwara's structure theorem of totally complex positive polarizations.

Eberhard Kaniuth : Power boundedness in Fourier and Fourier-Stieltjes algebras of locally compact Groups.

An element a of a Banach algebra A is called *power bounded* if $\sup\{\|a^n\| : n \in \mathbb{N}\} < \infty$. Elements with spectral radius $r(a) < 1$ are power bounded, and conversely the spectral radius of any power bounded element is ≤ 1. Accordingly, the algebra A is said to have the *power boundedness property* if every element with spectral radius ≤ 1 is actually power bounded.

Let G be a locally compact group, and let $B(G)$ denote the Fourier-Stieltjes algebra and $A(G)$ the Fourier algebra of G. Recall that when G is abelian, then $B(G)$ and $A(G)$ are isomorphic to $M(\widehat{G})$ and $L^1(\widehat{G})$, the measure algebra and the L^1-algebra of the dual group $\widehat{G}$ of G, respectively. This talk will mainly focus on a structure theorem for power bounded elements in $B(G)$ and the question of when the algebras $B(G)$ and $A(G)$ do have the power boundedness property. I will also touch the extension problem for power bounded functions and list some open problems.

Imed Kédim : On discontinuous subgroups acting on solvable homogeneous spaces.

Let G be a Lie group, H a closed subgroup of G and Γ a discrete subgroup of G. If the action of Γ on G/H is discontinuous, we consider the parameter space

$$R(\Gamma, G, H) := \left\{ \varphi \in \mathrm{Hom}(\Gamma, G) \;\middle|\; \begin{array}{l} \varphi \text{ is injective and } \varphi(\Gamma) \\ \text{is a discontinuous subgroup for } G/H \end{array} \right\}$$

The action of G on $\mathrm{Hom}(\Gamma, G)$ by composition on the left of the inner automorphisms leaves the parameter space invariant. The deformation space is the quotient space

$$\tau(\Gamma, G, H) = R(\Gamma, G, H)/G.$$

There are many natural questions about the structure and the topological and geometric features of $\tau(\Gamma, G, H)$. In this lecture, we present some recent developments, focusing on Hausdorffness and rigidity problems in the context of a solvable Lie groups.

Sami Kouki : Sur la restriction des séries discrètes de certains groupes résolubles algébriques.

Dans cet exposé, nous étudions la restriction des représentations, irréductibles et unitaires, de carré intégrable modulo le centre du groupe de Lie G produit semi-direct du groupe de Heisenberg par un tore maximal du groupe symplectique à un de ses sous-groupes algébriques connexes H. La détermination de la fonction de multiplicité est en rapport avec la conjecture de Guillemin-Sternberg connu sous le slogan "La quantification commute à la réduction". En effet, si $\pi \in \widehat{G}$ est une série discrète H-admissible, i.e. sa restriction Ã H est de la forme

$$\pi_{|H} = \sum_{\sigma \in \widehat{H}} m_\pi(\sigma)\sigma,$$

avec $m_\pi(\sigma) < \infty$, alors nous calculons cette multiplicité en terme de volume de variété réduite associée. Autrement dit, le principe "quantification commute à la réduction" reste valable dans ce cas.

Jean Ludwig : Intertwining operators for irreducible representations of exponential Lie groups.

Let G be a real solvable exponential Lie group with Lie algebra $\mathfrak{g}$ and let $f \in \mathfrak{g}^*$. We take two polarizations $\partial_j, j = 1, 2$, at f which meet the Pukanszky condition. Let $P_j := \exp \mathfrak{p}_j, j = 1, 2$, be the associated subgroups in G. The linear functional f defines unitary characters $\chi_f(\exp X) := e^{i\langle f, X\rangle}$, $X \in \mathfrak{p}_j$, of P_j. Let $\tau_j := \mathrm{ind}_{P_j}^G \chi_f$, $j = 1, 2$, be the corresponding induced representations, which are unitary and irreducible. It is well known that τ_1 and τ_2 are unitarily equivalent. The description of the intertwining operator of such an equivalence is given via an abstract integral $I_{\mathfrak{p}_2,\mathfrak{p}_1}$. The main problem is to show that this formal integral converges. In this talk we present a method, which has been developed in the early 90th by Hidenori, Didier Arnal and myself and published in 1996, to avoid this problem.

Nobuaki Obata : Spectral analysis of digraphs and coupled oscillators.

Spectrum of a graph or a digraph (directed graph) contains useful information for the analysis of (deterministic or stochastic) dynamics on it. We will report some new aspects of spectral analysis, in particular, of a Manhattan street network, i.e., a digraph where the underlying graph is the $n \times m$ lattice and each edge is given direction in such a way that left and right directed horizontal lines are placed alternately, and so are up and

down directed vertical lines. We will discuss some results on spectral analysis for the preliminary case of $2 \times n$ lattice and synchronization of coupled oscillators.

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[3] N. Obata and J. Rodriguez : in preparation (2013).

Christine Offerman : Multi-temporal Wave Equations on Riemannian Symmetric Spaces.

In 1976, Semenov, Tjan and Shansky introduced a formulation of the classical Cauchy problem for Riemannian symmetric spaces. In this case, the partial differential equation becomes a system of equations involving invariant differential operators on the symmetric space and multiple 'time' variables along with initial conditions involving the Weyl group harmonic polynomials. In 1999, Helgason studied thismulti-temporal systemon noncompact symmetric spaces using Fourier analysis. I will discuss recent results, obtained in collaboration with F. Gonzalez, for the cases of flat and compact symmetric spaces. I will begin by giving solution forms for the system involving the Fourier and Radon transforms. I will then talk about some properties of these solutions and the issue of uniqueness. In addition, I will present an energy form for this system as well as some of its properties, including its relation to the Fourier transform and a Plancherel-type result.

Takayuki Okuda : Classification of semisimple symmetric spaces with proper SL(2,R)-actions.

We give a complete classification of irreducible symmetric spaces for which there exist proper SL(2,R)-actions as isometries, using the criterion for proper actions by T. Kobayashi [Math. Ann. '89] and combinatorial techniques of nilpotent orbits. In particular, we classify irreducible symmetric spaces that admit surface groups as discontinuous groups, combining this with Y. Benoist's theorem [Ann. Math. '96].

Detlev Poguntke : Topology on the unitary dual of completely solvable Lie groups.

In his seminal doctoral dissertation (1962) A.A. Kirillov gave birth to the so-called orbit method in the case of a simply connected nilpotent Lie group G : The unitary dual $\hat{G}$ is in bijective correspondence with the orbit space $\mathfrak{g}^*/G$, where $\mathfrak{g}^*$ is the real linear dual of the Lie algebra $\mathfrak{g}$ of G, and G acts on $\mathfrak{g}^*$ via the coadjoint representation. This result was later extended to exponential Lie groups (i.e., the exponential map is a diffeomorphism), mainly by P. Bernat. It is natural to expect that these correspondences are actually diffeomorphisms, often called Kirillov conjecture. It took quite a while to establish this conjecture in the nilpotent case : I. Brown (1973), later K.I. Joy gave a different proof. Concerning the exponential case, in a monography (1994) H. Leptin and J. Ludwig showed that the Kirillov conjecture is here true as well. The splendid fundamental idea is the introduction of the notion of "Variable Groups", i.e., one considers families of (exponential) Lie groups, where the group law depends continuously on a parameter. Unfortunately, the mentioned proof is hard to comprehend, this applies in particular to the "philosophy" behind various cases and subcases. Even worse, there are errors in the proof, some of minor weight, but others are more serious : at least one case is missing. Those observations led me to a reconsideration of that proof. I think that in the (special) case of completely solvable Lie groups I can present a satisfactory picture (still, of course, resting on the notion of "Variable Groups") – and this will be the subject of my talk. The general case is "work in progress".

Dominique Manchon : The double shuffle structure of the Ohno-Okuda-Zudilin q-multiple zeta values.

Several q-analogues of multiple zeta values have been explored in the recent years. The model recently proposed by Y. Ohno, J.-I. Okuda and W. Zudilin shows particularly good algebraic properties, and readily extends to arguments running over all the integers, regardless to the sign. We exhibit the iterated Jackson integral representation of these qMZVs, and describe the q-double shuffle relations thus obtained. Renormalization at the limit q–>1 will also be addressed. Joint work with Jaime Castillo-Medina and Kurusch Ebrahimi-Fard.

Salah Mehdi : Representation theoretic differential operators.

We present several algebraic and geometric results on representations of Lie groups related to invariant differential operators. We will emphasis on Dirac operators in the context of coherent families of modules and translation functors.

Ayman Rahali : Contribution to the theory of unitary representations of Lie groups with co-compact nilradical. (Phd Thesis)

Let G be a locally compact group. By the unitary dual $\widehat{G}$ of G, we mean the set of all equivalence classes of irreducible unitary representations of G equipped with the Fell topology. The first representation-theoretic question concerning the group G is the description of the set $\widehat{G}$. Apart this question, a significant importance is attached to the determination of the topology of $\widehat{G}$. If G is a Lie group with Lie algebra $\mathfrak{g}$, then the investigation of the relationship between $\widehat{G}$ and the space $\mathfrak{g}^*/G$ of G-coadjoint orbits turns out to be a deep mathematical problem. In this direction, it is well-known that for a simply connected nilpotent Lie group or, more generally, for an exponential solvable Lie group G, the unitary dual $\widehat{G}$ is homeomorphic to the orbit space $\mathfrak{g}^*/G$.
In the present PhD thesis, first we consider (G, K) as a compact Riemannian symmetric pair, and let G_0 be the associated Cartan motion group. Under some assumptions on the pair (G, K), we give a precise description of the set $(\widehat{G_0})_{gen}$ of all equivalence classes of generic irreducible unitary representations of G_0. We also determine the topology of the space $(\mathfrak{g}_0^\ddagger/G_0)_{gen}$ of generic admissible coadjoint orbits of G_0 and we show that the bijection between $(\widehat{G_0})_{gen}$ and $(\mathfrak{g}_0^\ddagger/G_0)_{gen}$ is a homeomorphism. Furthermore, in the case where the pair (G, K) has rank one, we prove that the unitary dual $\widehat{G_0}$ is homeomorphic to the space $\mathfrak{g}_0^\ddagger/G_0$ of all admissible coadjoint orbits of G_0.
Also we describe the space of admissible coadjoint orbits of the Heisenberg motion group $G_n = \mathbb{T}^n \ltimes \mathbb{H}_n$, and we determine the topology of this space, where $\mathbb{H}_n$ is the $(2n + 1)$-dimensional Heisenberg group, and $\mathbb{T}^n$ is the n-dimensional torus acting on $\mathbb{H}_n$ by automorphisms. We show that the bijection between the unitary dual $\widehat{G_n}$ of G_n and its admissible coadjoint orbit space is a homeomorphism.
Finally, we show that every irreducible unitary representation of the Euclidean motion group $M_n = SO(n) \ltimes \mathbb{R}^n$, $n \geq 2$, is characterized by a particular element in its generalized moment set.

Atsumu Sasaki : A characterization of non-tube type Hermitian symmetric spaces by visible actions.

Our object of this talk is the complex manifolds which is the line bundle over the complexification of a non-tube type Hermitian symmetric space. This manifold is a homogeneous space but not a symmetric one. In this talk, we discuss a generalization of Cartan decomposition for this non-symmetric homogeneous space, in particular, the construction of an abelian part. Further, we explain that the above decomposition gives rise to a new example of visible actions.

Taro Yoshino : On Topological Blow-up.

Consider a Lie group G (or more generally, a topological group) acts continuously on a manifold M (or more generally, a locally compact Hausdorff space). The quotient space $X := G\backslash H$ is locally compact, but not always Hausdorff. In this talk, we introduce a method to understand the topology on such a non-Hausdorff space X. More precisely, for a given locally compact (not necessarily Hausdorff) space X, we construct a locally compact Hausdorff space Y, and a map $\tau : X \to 2^Y$. Then, the pair (Y, τ) has a complete information on the topology on X. In particular, (Y, τ) describes convergence of sequences or filters on X.