Entireness of L-functions of φ-sheaves on affine complete intersections
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Y. Taguchi and D. Wan[1]

Abstract. In a previous article [TW], the authors showed that the L-function attached to a π-adic φ-sheaf is meromorphic in a certain disk depending on the convergence condition of the φ-sheaf. That disk is in general best possible. The purpose of the present article is to show that for an affine complete intersection, either the L-function or its reciprocal is actually analytic (i.e. without poles) in the same disk.

1. Introduction

Let X be a complete intersection of equi-dimension n embedded in some smooth affine variety Y defined over a finite field $\mathbb{F}_q$ of q elements. Let $(\mathcal{E}, \varphi)$ be an α log-convergent π-adic φ-sheaf on X. In this paper, we prove that the L-function $L(\varphi/X, T)^{(-1)^{n-1}}$ is *analytic* (i.e. without poles) on a certain disk of "radius q^{α}". This result was conjectured in [TW].

As an application, we obtain the following result on L-functions of Drinfeld modules: Let $A = \mathbb{F}_q[t]$ and K its fraction field $\mathbb{F}_q(t)$. Let X be a variety over $\mathbb{F}_q$ and assume we are given an A-scheme structure $X \to \mathrm{Spec}A$. For a Drinfeld A-module ϕ over X with everywhere good reduction, its L-function $L(\phi/X, s)$ is defined ([G]). Here, $s = (z, y)$ is a variable ranging in the Goss complex plane $S_\infty = \mathbb{C}_\infty^\times \times \mathbb{Z}_p$. Recall ([TW]) that a Drinfeld module gives rise to an algebraic φ-sheaf $\mathcal{E}$ on X such that $L(\phi/X, s) = L(\mathcal{E} \otimes \mathcal{F}_y, z^{-1})$, where $\mathcal{F}_y$ is a family of overconvergent ∞-adic φ-sheaves parametrized by $y \in \mathbb{Z}_p$ (∞ is the place $(1/t)$ of K). Then our result implies that $L(\phi/X, s)^{(-1)^{n-1}}$ is entire (in the sense of [TW, §8]) on the whole plane S_∞ if X is a complete intersection of equi-dimension n in some smooth affine variety Y over $\mathbb{F}_q$. The entireness for this example was first conjectured by Goss in some cases ([G]).

The condition on X is always satisfied if X is an affine curve of equi-dimension over $\mathbb{F}_q$ by a well known theorem of Cowsik-Nori [CN].

Now we explain our results in more detail. Let $\mathbb{F}_q$ be a finite field of q elements with characteristic p. Let $X = \mathrm{Spec}R$ be an affine scheme of finite type over $\mathbb{F}_q$. For a field F containing $\mathbb{F}_q$, we write R_F for $R \otimes_{\mathbb{F}_q} F$. Let σ be the Frobenius endomorphism (q-th power map) $\otimes\, \mathrm{id}_F$ of R_F. Recall that an *algebraic φ-sheaf* $(\mathcal{E}, \varphi)$ on X over F (in the terminology of [TW]; or a *φ-module* over R_F in the terminology of [T]) is a pair consisting of a projective R_F-module $\mathcal{E}$ of finite rank together with a σ-semilinear map $\varphi : \mathcal{E} \to \mathcal{E}$ (or an R_F-linear map $\varphi : \sigma^*\mathcal{E} \to \mathcal{E}$, where $\sigma^*\mathcal{E} = \mathcal{E} \otimes_{R_F} R_F$ is the scalar extension by $\sigma : R_F \to R_F$). The L-function of a φ-sheaf $(\mathcal{E}, \varphi)$ on X is defined by the Euler product:

$$L(\varphi/X, T) \;=\; \prod_{x \in X_0} \det(1 - \varphi_x^{d(x)} T^{d(x)})^{-1} \qquad \in F[\![T]\!],$$

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where X_0 is the set of closed points of X, φ_x is the endomorphism induced by φ on the stalk $\mathcal{E}_x$ at the closed point x and $d(x)$ is the degree $[\mathbb{F}_q(x) : \mathbb{F}_q]$ of the closed point x.

If F is a complete discrete valuation field with a uniformizer π, let $\hat{R_F}$ be the π-adic completion of R_F. By replacing R_F in the above by $\hat{R_F}$, we obtain the notion of a *π-adic φ-sheaf* on X, for which the above definition of the L-function remains valid. We consider the L-function $L(\varphi/X, T)$ as a function on the completion $\mathbb{C}_\pi$ of an algebraic closure of F. It is said to be *analytic* on a subset D if it converges at all points in D. For a π-adic φ-sheaf, we consider certain convergence conditions, such as "overconvergent" and "α log-convergent" (see §4 and [TW, §3]). Then our main theorem is that, if $(\mathcal{E}, \varphi)$ is an α log-convergent π-adic φ-sheaf on an affine scheme X satisfying reasonable conditions as stated at the beginning, then $L(\varphi/X, T)^{(-1)^{n-1}}$ is analytic in the open disk $\mathrm{ord}_\pi(T) > -\alpha$.

The idea of our proof is as follows. It is shown in [TW] that the L-function $L(\varphi/X, T)$ is a rational (resp. meromorphic) function in T if φ is algebraic (resp. overconvergent). If X is the affine or toric n-space, the characteristic p version of the Dwork trace formula shows that $L(\varphi/X, T)^{(-1)^{n-1}}$ is actually a polynomial (resp. analytic) if φ is algebraic (resp. overconvergent)(see [W1]). More generally, Anderson [A] has recently obtained an explicit characteristic p version of the Monsky trace formula, which shows that $L(\varphi/X, T)^{(-1)^{n-1}}$ for an algebraic φ is also a polynomial if X is smooth affine of equi-dimension n. Then our strategy is to *truncate* the α log-convergent π-adic φ-sheaf $(\mathcal{E}, \varphi)$ to obtain a sequence of algebraic φ-sheaves $(\mathcal{E}_j, \varphi_j)$, to which Anderson's theorem applies. Thus each $L(\varphi_j/X, T)^{(-1)^{n-1}}$ is a polynominal. By the uniform variation result of L-functions ([TW, §5]), we can pass to the limit $L(\varphi/X, T) = \lim_j L(\varphi_j/X, T)$ as long as we stay in the disk $\mathrm{ord}_\pi(T) > -\alpha$, and our result follows (§4). (In the special case that the π-adic φ-sheaf is overconvergent or more generally ∞ log-convergent, the proof in the algebraic case should carry over since the resulting operators acting on various Banach spaces are compact and thus Serre's spectral theory applies. However, if the π-adic φ-sheaf is α log-convergent with $\alpha < \infty$, then the resulting operators are not compact in general and thus Serre's spectral theory cannot be used. One could try to prove a weaker version of the spectral theory to make the proof work but this would be a little complicated. We shall instead use the uniform variation result of L-functions. This approach is significantly simpler.)

As for the generalization of the base scheme X to the case of a set-theoretic complete intersection of equi-dimension n in some smooth affine scheme Y of finite type over $\mathbb{F}_q$, the idea is to use the inclusion-exclusion principle as in [TW] to express the L-function over X as an alternating product of L-functions over the smooth ambient space Y. One then shows that this gives rise to a homological formula in terms of Koszul complex. The acyclicity of this Koszul complex is implied by the assumption that X is a complete intersection.

In the special case where X is an affine complete intersection (i.e., when Y is an affine space), our original proof uses a rather simple characteristic p version of the Dwork trace formula. Once Anderson's formula becomes available for smooth affine Y, our proof carries over immediately to complete intersections in Y. Thanks to Anderson's elegant and elementary proof in the smooth case, the proof of our result in the more general case is also quite elementary and explicit.

For algebraic φ-sheaves, another possible approach, entirely cohomological and much less elementary, would be to use the Katz-Deligne's congruence formula [K][D] for L-functions in terms of étale cohomology with compact support. The desired entireness could then be deduced using Poincaré duality, Lefschetz affine theorem and the excision sequence. See [W2] for a related ℓ-adic proof for the entireness of classical characteristic zero L-functions.

2. Algebraic φ-sheaves over smooth base

In this section, we review Anderson's theorem [A] about entireness of L-functions for algebraic φ-sheaves when the base scheme $X = Y = \mathrm{Spec}R$ is smooth and equi-dimensional. Let Ω_R (resp. $\Omega_{R_F/F}$) be the module of Kähler differentials of R (resp. of R_F relative to F). By our assumption of smoothness and equi-dimensionality, the module Ω_R (resp. $\Omega_{R_F/F}$) is a projective R-module (resp. R_F-module) of rank equal to the Krull dimension of R. Set

$$\omega_{R,F} \;=\; (\det_R \Omega_R) \otimes_{\mathbb{F}_q} F \;=\; \det_{R_F} \Omega_{R_F/F}.$$

For a φ-sheaf $(\mathcal{E}, \varphi)$ on X, we define its *adjoint* $(\check{\mathcal{E}}, \check{\varphi})$ by setting

$$\begin{cases} \check{\mathcal{E}} &= \mathrm{Hom}_{R_F}(\mathcal{E}, \omega_{R,F}) \\ \check{\varphi}(\check{e}) &= C_\sigma \circ \check{e} \circ \varphi \qquad\qquad \text{for } \check{e} \in \check{\mathcal{E}}. \end{cases}$$

Here, $C_\sigma : \omega_{R,F} \to \omega_{R,F}$ is the Cartier operator relative to F; thus it is a Cartier linear map, i.e., one has $C_\sigma(f^\sigma w) = f C_\sigma(w)$ for any $f \in R_F$ and $w \in \omega_{R,F}$. Note that $\check{\varphi}$ is also Cartier linear.

Let $C : \mathcal{E} \to \mathcal{E}$ be an F-linear endomorphism of a (possibly infinite-dimensional) vector space. A *nucleus* $\mathcal{E}_0$ for C is a finite-dimensional F-subspace of $\mathcal{E}$ such that $\mathcal{E} = \cup_{n\geq 0} C^{-n}(\mathcal{E}_0)$. If C has a nucleus, then the Fredholm determinant of C,

$$\det(1 - TC) \;:=\; \det(1 - T(C|\mathcal{E}_0)) \qquad \in F[T],$$

is defined and independent of the choice of a nucleus $\mathcal{E}_0$.

If $\mathcal{E}$ is an R_F-module, any Cartier linear map $C : \mathcal{E} \to \mathcal{E}$ is of trace class, and hence has a nucleus ([A], 2.2). Thus the Fredholm determinant

$$\det(1 - T\check{\varphi}) \qquad \in F[T]$$

is defined for the adjoint $(\check{\mathcal{E}}, \check{\varphi})$ of the φ-module $(\mathcal{E}, \varphi)$ and it is a polynomial in T. One of the main results of [A] is:

Theorem 2.1. (Anderson) *If $Y = \mathrm{Spec}R$ is smooth of equi-dimension N and $(\mathcal{E}, \varphi)$ is an algebraic φ-sheaf on Y, then*

$$L(\varphi/Y, T)^{(-1)^{N-1}} \;=\; \det(1 - T\check{\varphi}).$$

In particular, the L-function $L(\varphi/Y, T)^{(-1)^{N-1}}$ is a polynomial in T.

(In [A], only the case $F = \mathbb{F}_q$ is treated, but the proof is valid for any field F containing $\mathbb{F}_q$.)

This is a kind of Dwork trace formula, and the special case for tori $Y = \mathbb{G}_m^N$ (and also for the affine N-space $Y = \mathbb{A}^N$) is proved in [W1, Lemma 4.2]. In the torus case, the proof consists of more or less straightforward computations. In the general case, Anderson employs the full generality of the formalism of Frobenius- and Cartier-modules to reduce the proof to the case where Y has no (or few) $\mathbb{F}_{q^n}$-valued points for any given n.

3. Entireness of L-functions in the algebraic case

Let $Y = \mathrm{Spec}R$ be a regular affine scheme of finite type over $\mathbb{F}_q$, with equi-dimension N, as in the previous section. Let $X = \mathrm{Spec}(R/I)$ be a closed subscheme of Y, where I is an ideal of R. In this section, we use Anderson's theorem to show that the entireness of L-functions holds for algebraic φ-sheaves on X provided that X is a set-theoretic complete intersection in Y.

Assume that the ideal I is generated by a set of elements $\{f_1, \cdots, f_m\}$ of R. We want to derive a homological formula for the L-function $L(\varphi/X, T)$. The assumption that X is a complete intersection in Y (i.e., that $m = \dim Y - \dim X$) will be made only at the end of this section.

Let F be any field containing $\mathbb{F}_q$, and let $(\mathcal{E}, \varphi)$ be a φ-sheaf on X over F. By Trick (2.2) of [TW], we may and do assume $\mathcal{E}$ is a free $(R/I) \otimes_{\mathbb{F}_q} F$-module of finite rank. Choose a lifting of the φ-sheaf $(\mathcal{E}, \varphi)_{/X}$ to a φ-sheaf on Y. Namely, one simply lifts the entries of the matrix of φ (which are in R/I) to elements in R. Denote the lifted φ-sheaf by $(\mathcal{E}, \varphi)_{/Y}$, or simply by $(\mathcal{E}, \varphi)$. Thus, $\mathcal{E}$ becomes a free R_F-module of finite rank. For each subset $S \subset \{1, \cdots, m\}$, put $f_S = \prod_{i \in S} f_i$ ($f_\phi = 1$ if ϕ is the empty set) and define a φ-sheaf $(\mathcal{E}, \varphi_S)$ on Y by setting

$$\varphi_S \;=\; f_S^{q-1} \circ \varphi$$

(so $\varphi_\phi = \varphi$), where the map f_S means multiplication by f_S. The inclusion-exclusion argument used in the proof of Theorem 4.1 of [TW] gives the formula

$$L(\varphi/X, T) \;=\; \prod_{i=0}^{m} \prod_{|S|=i} L(\varphi_S/Y, T)^{(-1)^i}, \tag{3.1}$$

where the second product is over all subsets S of $\{1, \cdots, m\}$ with cardinality i.

Note that each factor on the right side of (3.1) is an L-function of an algebraic φ-sheaf on the affine smooth equi-dimensional scheme Y. Thus, Anderson's theorem can be applied to each such factor. For this purpose, let $(\check{\mathcal{E}}, \check{\varphi}_S)$ be the adjoint of $(\mathcal{E}, \varphi_S)$ over Y. Then we have $\check{\varphi}_S = \check{\varphi} \circ f_S^{q-1}$. By Theorem 2.1,

$$L(\varphi_S/Y, T)^{(-1)^{N-1}} \;=\; \det(1 - T\check{\varphi}_S | \check{\mathcal{E}}).$$

This together with (3.1) shows

$$L(\varphi/X, T)^{(-1)^{N-m-1}} \;=\; \prod_{i=0}^{m} \prod_{|S|=i} \det(1 - T\check{\varphi}_S | \check{\mathcal{E}})^{(-1)^{m-i}}. \tag{3.2}$$

Since $\mathcal{E}$ is a free R_F-module of finite rank, the adjoint $\check{\mathcal{E}}$ is a projective R_F-module. By applying Trick (2.2) of [TW] if necessary, we may assume that $\check{\mathcal{E}}$ is a free R_F-module. The elements f_i in R act on $\check{\mathcal{E}}$ by multiplication. Let $K_{.}(\check{\mathcal{E}})$ be the Koszul complex on $\check{\mathcal{E}}$ defined by the commuting operators f_i $(i = 1, \cdots, m)$. Namely, for $0 \le i \le m$, we set

$$K_i(\check{\mathcal{E}}) \;=\; \bigoplus_{|S|=i} \check{\mathcal{E}} \cdot e_S,$$

where $e_S = e_{j_1} \wedge \cdots \wedge e_{j_i}$ if $S = \{j_1, \cdots, j_i\}$ and $j_1 < \cdots < j_i$. Set $K_i(\check{\mathcal{E}}) = 0$ if $i < 0$ or $i > m$. The boundary operator $\partial : K_{i+1}(\check{\mathcal{E}}) \to K_i(\check{\mathcal{E}})$ (for $0 \le i < m$) is given by:

$$\partial(e_{j_1} \wedge \cdots \wedge e_{j_{i+1}}) \;=\; \sum_{k=1}^{i+1} (-1)^{k+1} f_{j_k} (e_{j_1} \wedge \cdots \wedge e_{j_{k-1}} \wedge e_{j_{k+1}} \wedge \cdots \wedge e_{j_{i+1}}).$$

Define

$$\check{\varphi}_i \;=\; \bigoplus_{|S|=i} \check{\varphi}_{S^c}, \qquad \check{\varphi}_. \;=\; \bigoplus_i \check{\varphi}_i,$$

where S^c denotes the compliment of S, namely, $S^c = \{1, \cdots, m\} - S$. All of these operators are of trace class. Formula (3.2) can now be rewritten as

$$L(\varphi/X, T)^{(-1)^{N-m-1}} \;=\; \prod_{i=0}^{m} \det(1 - T\check{\varphi}_i | K_i(\check{\mathcal{E}}))^{(-1)^i}. \tag{3.3}$$

It is straightforward to check that $\check{\varphi}_.$ is actually a homomorphism of complexes, i.e., the diagram

$$\begin{array}{ccc} K_{i+1}(\check{\mathcal{E}}) & \xrightarrow{\partial} & K_i(\check{\mathcal{E}}) \\ {\scriptstyle \check{\varphi}_{i+1}} \downarrow & & \downarrow {\scriptstyle \check{\varphi}_i} \\ K_{i+1}(\check{\mathcal{E}}) & \xrightarrow{\partial} & K_i(\check{\mathcal{E}}) \end{array}$$

commutes. Let $H_i(\check{\mathcal{E}})$ denote the i-th homology group of the complex $(K.(\check{\mathcal{E}}), \partial)$. The map $\check{\varphi}_i$ on $K_i(\check{\mathcal{E}})$ induces a map $\check{\varphi}_i$ on $H_i(\check{\mathcal{E}})$. Passing to the homology level, (3.3) gives the formula:

$$L(\varphi/X, T)^{(-1)^{N-m-1}} \;=\; \prod_{i=0}^{m} \det(1 - T\check{\varphi}_i | H_i(\check{\mathcal{E}}))^{(-1)^i}. \tag{3.4}$$

Theorem 3.1. *Assume $X = \mathrm{Spec}(R/I)$ is an equi-dimensional complete intersection in some regular affine scheme $Y = \mathrm{Spec}R$ of finite type over $\mathbb{F}_q$. Let $(\mathcal{E}, \varphi)$ be an algebraic φ-sheaf on X over F. If $\dim X = n$, then we have*

$$L(\varphi/X, T)^{(-1)^{n-1}} \;=\; \det(1 - T\check{\varphi}_0 | H_0(\check{\mathcal{E}})). \tag{3.5}$$

In particular, the L-function $L(\varphi/X, T)^{(-1)^{n-1}}$ is a polynomial in T.

Proof. Since R is regular and thus Cohen-Macaulay, the unmixedness theorem shows that I is generated by a regular sequence $\{f_1, \cdots, f_m\}$ with $m = \dim Y - \dim X$. Furthermore, each prime ideal in $\mathrm{Ass}(R/I)$ has the same height m. Since the space X is equi-dimensional, we may assume that Y is also equi-dimensional by dropping lower dimensional components of Y. Now, we can apply the above argument and formula (3.4).

Since $\check{\mathcal{E}}$ is a projective R_F-module, the sequence $\{f_1, \cdots, f_m\}$ also forms a regular sequence for $\check{\mathcal{E}}$. This implies

$$\begin{cases} H_i(\check{\mathcal{E}}) &= 0, & \text{if } i \neq 0, \\ H_0(\check{\mathcal{E}}) &= \check{\mathcal{E}}/(f_1, \cdots, f_m)\check{\mathcal{E}}. \end{cases}$$

Since $N - m - 1 = \dim Y - (\dim Y - \dim X) - 1 = \dim X - 1$, the proof is complete.

4. Entireness of L-functions in the π-adic case

We first recall the definition of $\alpha \log$-convergent π-adic φ-sheaves on an affine scheme $X = \mathrm{Spec}R$, where R is an $\mathbb{F}_q$-algebra of finite type, not necessarily smooth. Let K_π be a complete discrete valuation field containing $\mathbb{F}_q$, with a uniformizer π. Let $R_{K_\pi} = R \otimes_{\mathbb{F}_q} K_\pi$ and let $\hat{R_{K_\pi}}$ be the completion of R_{K_π} with respect to the π-adic topology. Let σ be the Frobenius endomorphism (q-th power map)$\hat{\otimes}\mathrm{id}_{K_\pi}$ of $\hat{R_{K_\pi}}$. Recall that a *π-adic φ-sheaf* $(\mathcal{E}, \varphi)$ on X is a pair consisting of a projective $\hat{R_{K_\pi}}$-module $\mathcal{E}$ of finite rank together with a σ-semilinear map $\varphi : \mathcal{E} \to \mathcal{E}$. The L-function of a π-adic φ-sheaf $(\mathcal{E}, \varphi)$ on X is defined by the Euler product:

$$L(\varphi/X, T) \ = \ \prod_{x \in X_0} \det(1 - \varphi_x^{d(x)} T^{d(x)})^{-1} \qquad \in K_\pi[\![T]\!],$$

where, as before, X_0 is the set of closed points of $X = \mathrm{Spec}R$, φ_x is the endomorphism induced by φ on the stalk $\mathcal{E}_x$ at the closed point x and $d(x)$ is the degree of the closed point x over $\mathbb{F}_q$.

Choose a finite set of generators $\{x_1, \cdots, x_d\}$ of R as an $\mathbb{F}_q$-algebra. Any element of $\hat{R_{K_\pi}}$ can be written (non-uniquely) in the form

$$\text{(4.1)} \qquad \sum_k c_k x^k \qquad \text{with} \ \ c_k \in K_\pi \ \ \text{and} \ \ c_k \to 0 \ (|k| \to \infty),$$

where $k = (k_1, \cdots, k_d)$ is a multi-index with $k_i \geq 0$, $x^k = x_1^{k_1} \cdots x_d^{k_d}$ and $|k| = k_1 + \cdots + k_d$.

Definition. (cf. [TW], §3) (1) Let α be a non-negative real number. An element of $\hat{R_{K_\pi}}$ is said to be $\alpha \log$-*convergent* if it can be expressed as in (4.1) with

$$\text{(4.2)} \qquad \liminf_{|k| \to \infty} \frac{\mathrm{ord}_\pi c_k}{\log_q |k|} \ \geq \ \alpha.$$

An element is said to be $\infty \log$-*convergent* if it is $\alpha \log$-convergent for all $\alpha \geq 0$.

(2) A π-adic φ-sheaf $(\mathcal{E}, \varphi)$ is said to be $\alpha \log$-*convergent* if there is an open covering of X such that, on each affine open piece, $\mathcal{E}$ is free and φ is represented by a matrix whose entries are all $\alpha \log$-convergent.

Theorem 4.1. *Assume X is an equi-dimensional complete intersection in some regular affine scheme Y of finite type over $\mathbb{F}_q$. Let $(\mathcal{E}, \varphi)$ be an α log-convergent π-adic φ-sheaf on X. If $\dim X = n$, then the L-function $L(\varphi/X, T)^{(-1)^{n-1}}$ is analytic (i.e. has no poles) in the open disk $\mathrm{ord}_\pi(T) > -\alpha$. In particular, it is entire (i.e. analytic on the whole $\mathbb{C}_\pi$) if $(\mathcal{E}, \varphi)$ is overconvergent.*

Proof. It is known from [TW] that $L(\varphi/X, T)^{(-1)^{n-1}}$ is meromorphic in the open disk $\mathrm{ord}_\pi(T) > -\alpha$. We need to prove that it has no poles in the same disk.

Again, by Trick (2.2) of [TW], we may assume that $\mathcal{E}$ is a free $\hat{R_{K_\pi}}$-module of rank r for some $r \geq 0$. The matrix of φ (with respect to a fixed basis of $\mathcal{E}$), still denoted by φ, can be written as a power series

$$\varphi = \sum_k M_k x^k,$$

where M_k are $r \times r$ matrices with coefficients in K_π. Since φ is α log-convergent, we can find an expression as above for which

$$\liminf_{|k| \to \infty} \frac{\mathrm{ord}_\pi M_k}{\log_q |k|} \geq \alpha. \qquad (4.3)$$

Now, for each non-negative integer j, let φ_j be the truncation

$$\varphi_j = \sum_{|k| \leq j} M_k x^k.$$

It is clear that $(\mathcal{E}, \varphi_j)$ defines an algebraic φ-sheaf on X for each $j \in \mathbb{N}$.

Define a topology on the set $\mathbb{N} \cup \{\infty\}$ by declaring its proper closed sets to be exactly the subsets of finite cardinality not containing ∞. The space $\mathbb{N} \cup \{\infty\}$ is then compact with only one limit point ∞. For each $\epsilon > 0$, it is easy to check that the family $(\mathcal{E}, \varphi_j)$ defines a continuous family of uniformly $(\alpha - \epsilon)$ log-convergent φ-sheaves parametrized by $\mathbb{N} \cup \{\infty\}$, in the sense of [TW], where φ_∞ is defined to be φ. By Theorem 5.2 of [TW], the family $L(\varphi_j/X, T)$ is an $(\alpha - \epsilon)$-continuous family of meromorphic functions on the open disk $\mathrm{ord}_\pi(T) > -(\alpha - 2\epsilon)$. By Theorem 3.1, each L-function $L(\varphi_j/X, T)^{(-1)^{n-1}}$ for algebraic φ_j $(j < \infty)$ has no poles on the open disk $\mathrm{ord}_\pi(T) > -(\alpha - 2\epsilon)$. Since ∞ is the limit point of the space $\mathbb{N} \cup \{\infty\}$, it follows from the uniform variation of zeroes and poles that $L(\varphi/X, T)^{(-1)^{n-1}}$ has also no poles on the open disk $\mathrm{ord}_\pi(T) > -(\alpha - 2\epsilon)$. The proof is complete.

Remark. In [TW], parameter spaces are taken to be compact subsets of the p-adic integers $\mathbb{Z}_p$. The same proof works for any compact topological parameter space. Or one can simply choose an infinite p-adic convergent subsequence $\{j_i\}$ of $\{j\}$ and use the compact parameter space $\{j_i\} \cup \{\lim_{i \to \infty} j_i\}$ contained in $\mathbb{Z}_p$.

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Yuichiro Taguchi
Tokyo Metropolitan University, Hachioji, Tokyo 192-03, JAPAN
taguchi@math.metro-u.ac.jp

Daqing Wan
Pennsylvania State University, University Park, PA16802
wan@math.psu.edu