Novas Aplicações das Matemáticas na Indústria

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Institute of Mathematics for Industry
Kyushu University
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Fukuoka

Math-for-industry
Education & Research Hub

Institute of Mathematics for Industry
Kyushu University

Contents

Mathematics – Japanese Background

Policy Study in 2006 by the Japanese Government

Europe and USA:
65% of workers in R&D departments in private companies have **Mathematics** as background.
Japan: only 26%
This shortage and nearly 40% gap must be overcome.

In Japan, **Pure Mathematics** have been studied much more than **Applied Mathematics**.

What is Mathematics-for-Industry ?

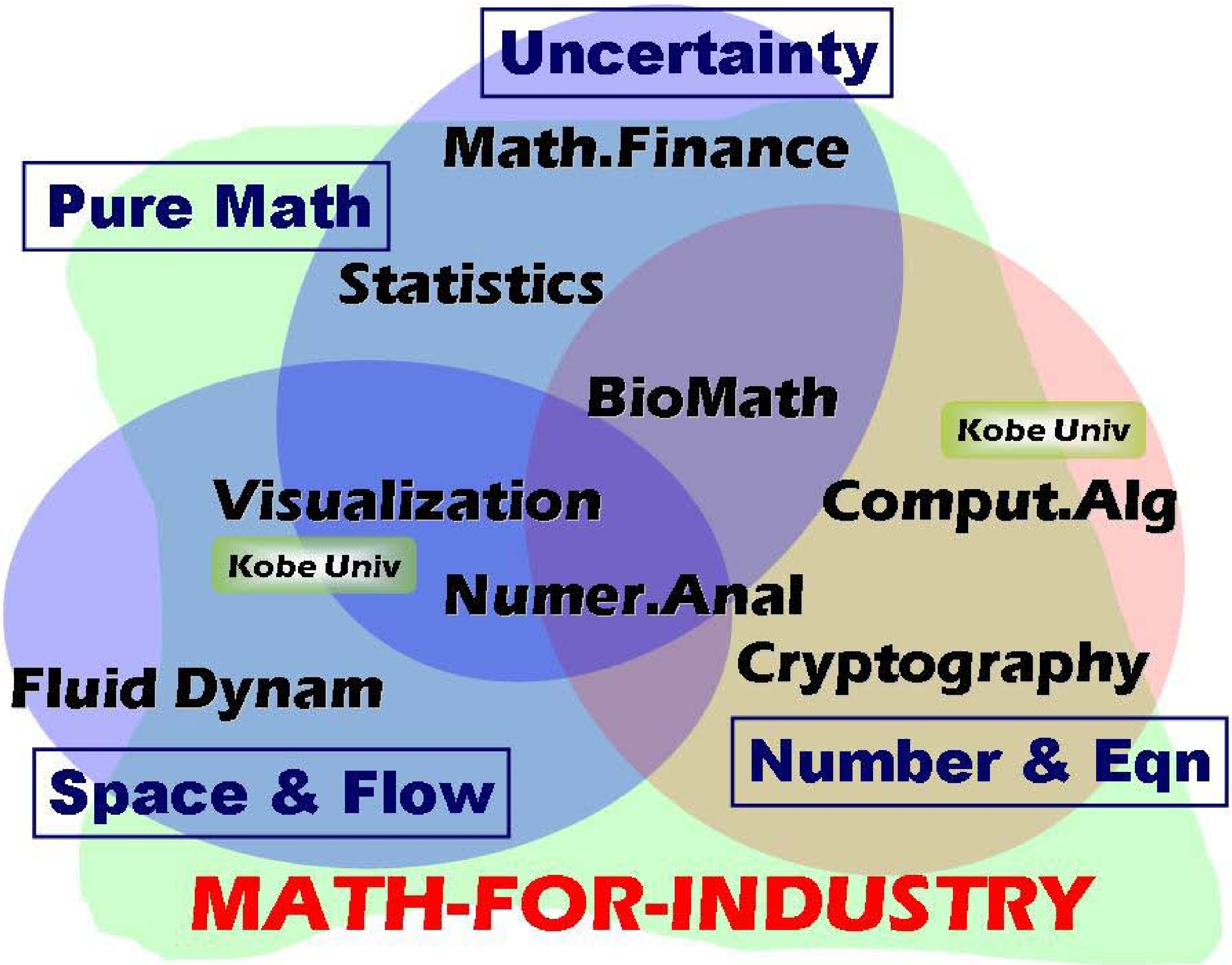
- Mathematics-for-Industry (MI, for short) is a **new research area** that will provide a foundation for creating future technologies. MI responds to the industrial needs by reorganizing and merging **pure and applied mathematics**.
- Main purpose of our program is to perform the **education** and **research** activities in MI.

http://gcoe-mi.jp/

Global Center Of Excellence Program (like CEPID-FAPESP)
April 2008- March 2013

The concentrated-funding-in Global COE Program (FY2008) supported by the MEXT

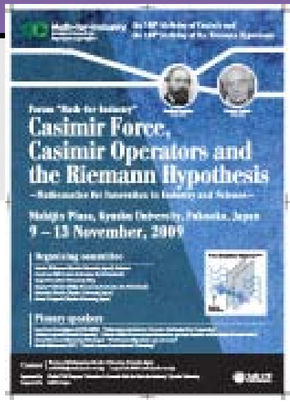
Disciplines covered by MI



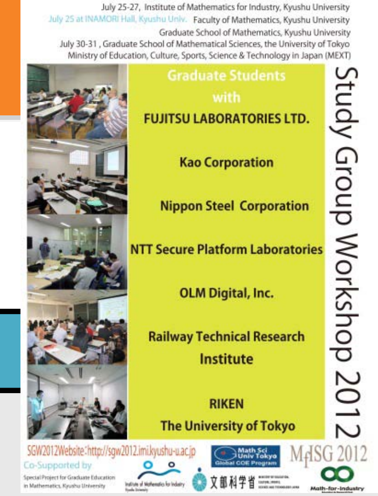
Forum "Math-for-Industry"

IMI founded

FMI 2009	Fukuoka	November 9-13	Casimir Force, Casimir Operators and the Riemann Hypothesis
FMI 2010	Fukuoka		October 21-23 Information Security, Visualization, and Inverse Problems, on the basis of Optimization Techniques
FMI 2011	Honolulu	October 24- 28	TSUNAMI - Mathematical Modelling - Using Mathematics for Natural Disaster Prediction, Recovery and Provision for the Future -
FMI 2012	Fukuoka		October 22-26 Information Recovery and Discovery



Study Group Workshops (2010, 2011, 2012)



3rd SGW: July 25-27 & 30-31, 2012
Kyushu University & University of Tokyo

Company	Subject
NTT Secure Platform Laboratories	**Arithmetic-Cryptography**
FUJITSU LABORATORIES LTD.	Integer factorization problems **Arithmetic-Cryptography**
Kao Corporation	**Graph Theory-Optimization**
OLM Digital, Inc.	CG of animation **Geometry and Statistics**
Nippon Steel Corporation	Multi-scale modeling, Anomalous diffusion **Geometry-Topology-Probability-PDE**
Railway Technical Research Institute	Space curves **Differential Geometry**
RIKEN / The University of Tokyo	**Mathematical Physics**

Institute of Mathematics for Industry (IMI)

First Industrial Math. Institute in Japan, Founded in April 2011

Institute of Mathematics for Industry
Kyushu University

Organization of IMI (27 full-time staffs)

◎ **Advanced Math. Technology Section**

◎ **Applied Math. Research Section**

◎ **Fundamental Math. Section**

Researchers of pure math., interested in application

◎ **Laboratory of Advanced Software in Mathematics**

◎ **Visitor Section** (from Academia & Industry)

Nikkei Newspaper (June 2010)

Nishinippon Newspaper (Feb. 2010)

R&D in Companies

Carrier Pass for PhD increased

University, etc.

PhD numbers increased (new system for degree)

Joint research with companies

Team Research

Doctoral course (Master course)

- **Experiencing joint research**
- **Human exchange**

Joint Projects（no joint projects before 2005）

ASAHI GLASS	FUJITSU	NIPPON STEEL CORP.
PANASONIC	Nisshin Fire & Marine Insurance Co., Ltd.	Mitsubishi Research Inst. Inc.
MAZDA MOTOR CORP.	KDDI	HITACHI
IBM Japan	ETRI, Korea	OLM Digital, Inc. & WETA Digital, New Zealand (CREST)
Studio Phones	Nonprofit organization--Science Accessibility Net	
NTT	12R, Singapore	Appeared from long-term internship
New Energy and Industrial Technology Development Organization	National Institute of Information and Communications Technology	Progressing now

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I participate here !

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Journal of Math-for-Industry

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Editor-in-Chief: Masato Wakayama
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Math-for-industry
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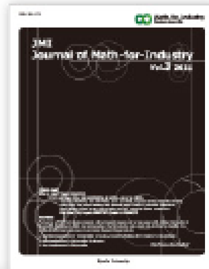
- Vol.4
- Vol.3
- Vol.2
- Vol.1

Editorial Board

Guide for Authors



The Journal of Math-for-Industry presents original research papers and survey papers with original view points in all scientific disciplines in which the mathematics or that in industry play a basic role. Articles by scientists in a variety of interdisciplinary areas are published.

Research Areas Include:

1. Significant applications of mathematics to industry, including feedback from industry to mathematics
2. New developments in Mathematics for industry
3. New developments in Mathematics

Access Full Text

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Guide for Authors

What's new JMI2012B has been published! (5 October 2012)

Institute of Mathematics for Industry & Faculty of Mathematics, Kyushu University

We are waiting for your submission to JMI!

II. Application of Singularity Theory to Visualization of Big Data

Osamu Saeki (IMI, Kyushu Univ.)

Joint work with Shigeo Takahashi (Univ. of Tokyo)

November 28, 2012

§1. Visualization of Scalar Function Data

Level set

M^n : differentiable manifold of dimension n (or a region in $\mathbf{R}^n$)

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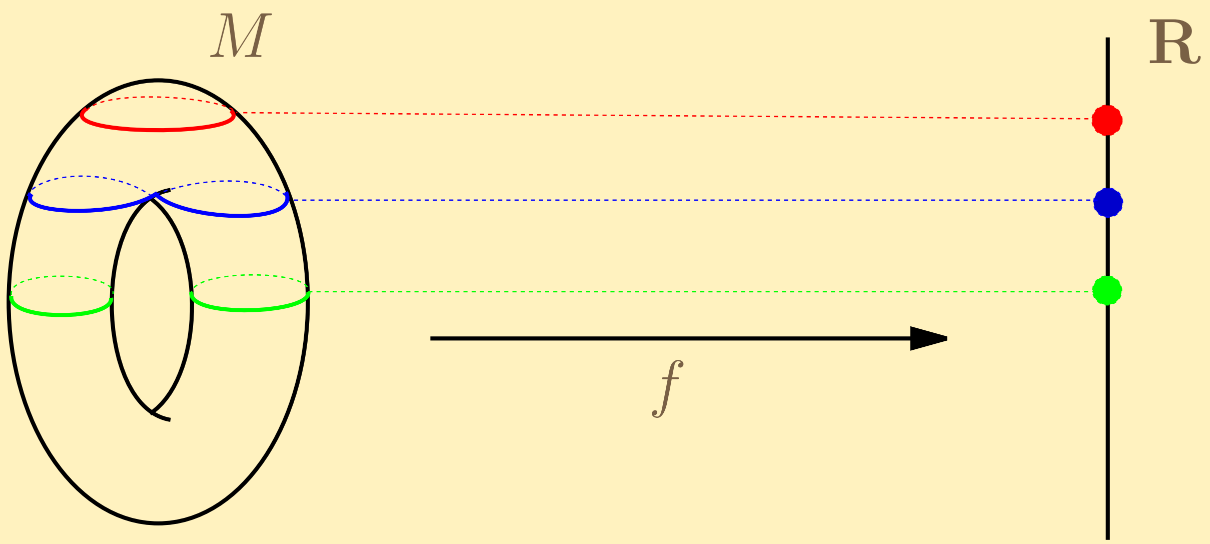
Example 1.2 Altitude from the sea level (height function):
level set $=$ contour line

Example of level sets



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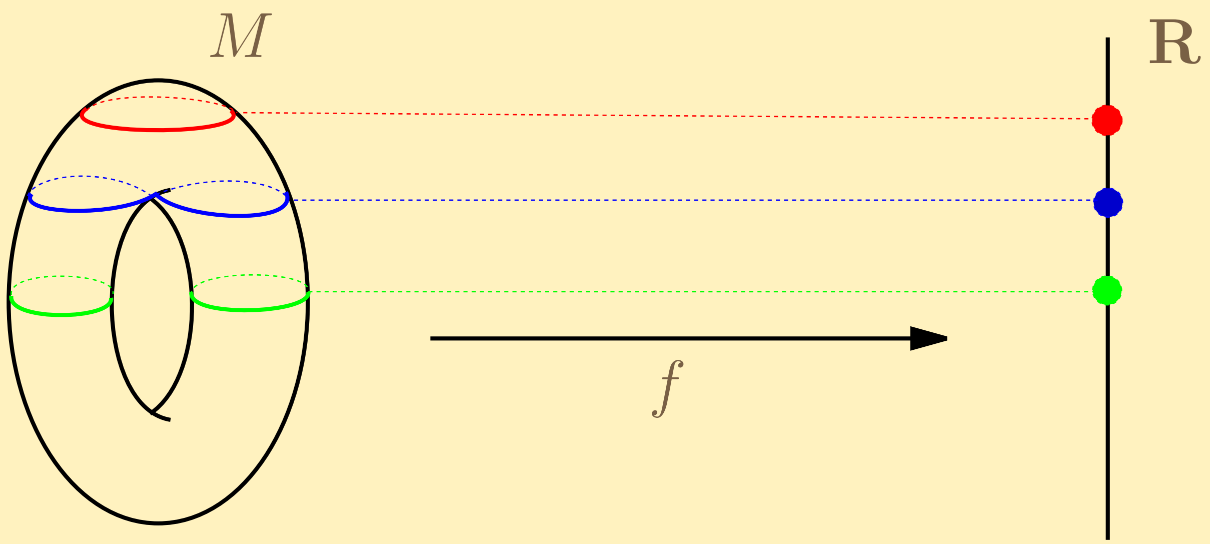

A level set may not be connected.

Reeb graph

The space (or graph) obtained by contracting each connected component of the level set to a point is called a **Reeb graph** (or contour tree, volume skeleton tree, Stein factorization, ...).

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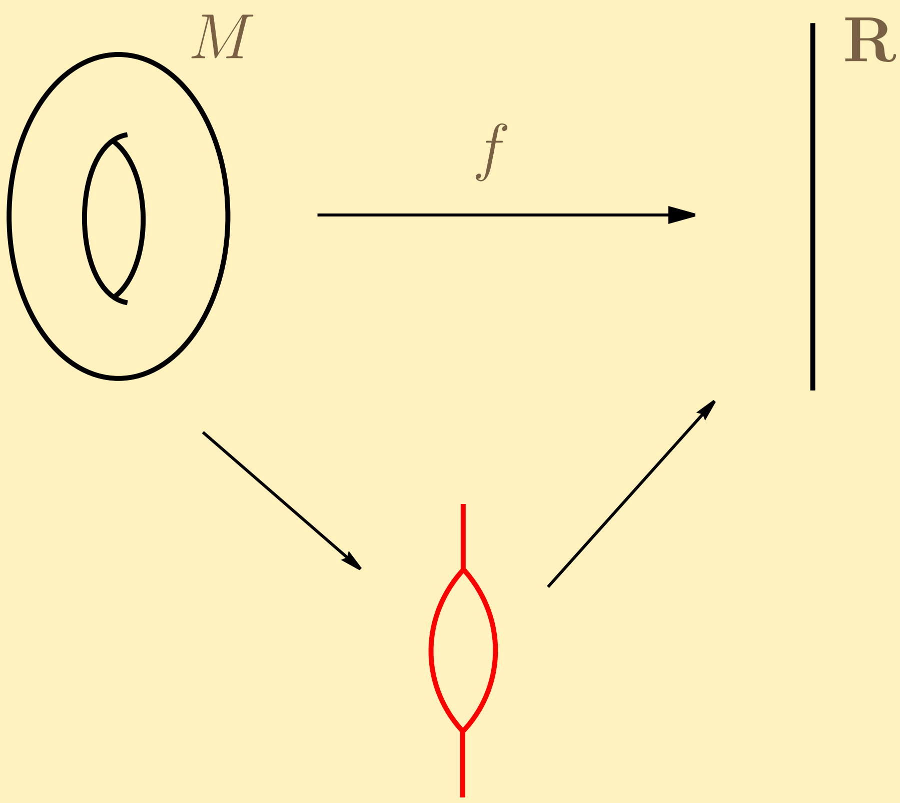

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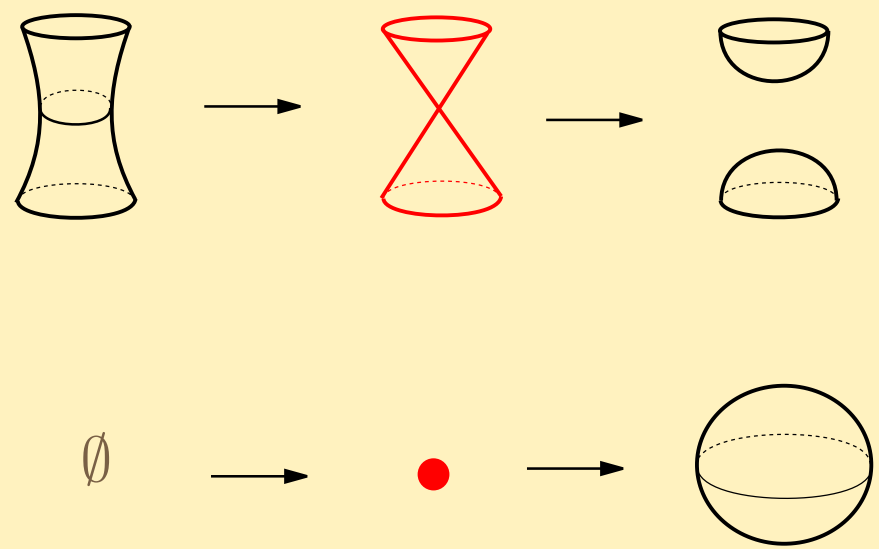


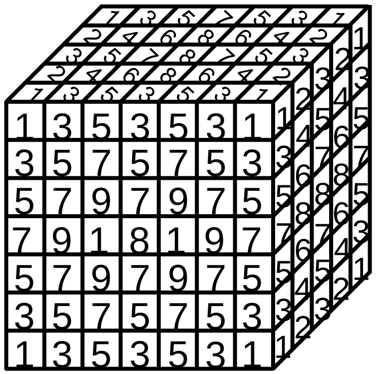
Example of level-surface change for a 3-dimensional scalar function

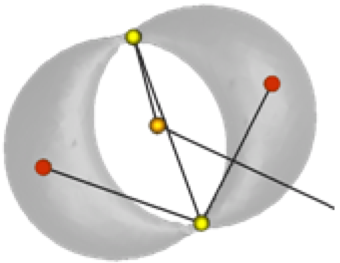
Direct Volume Rendering

- Big Size
- Complicated Structure
- Noise

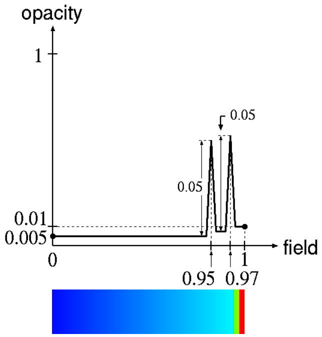
Volume Data → Analysis → Transfer Function Design → Rendering

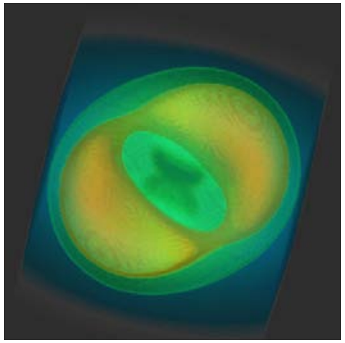
Visual Feedback





Reeb graph

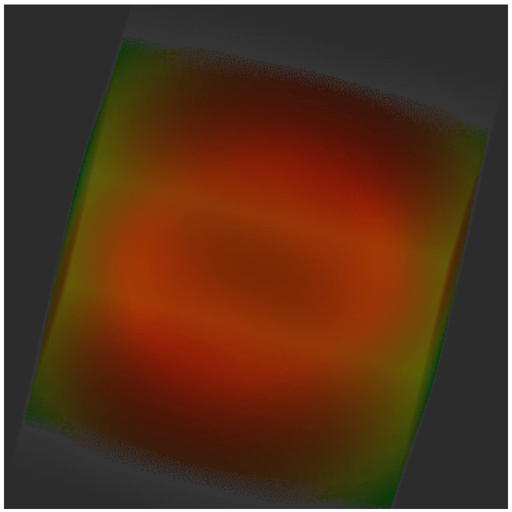


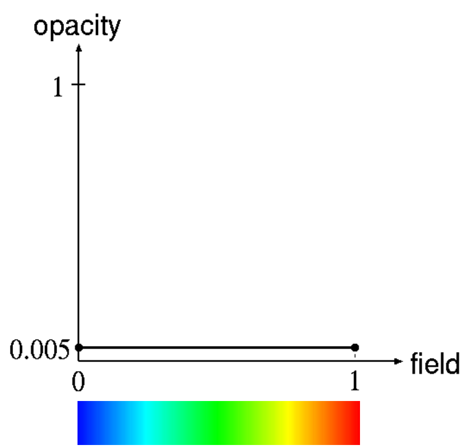


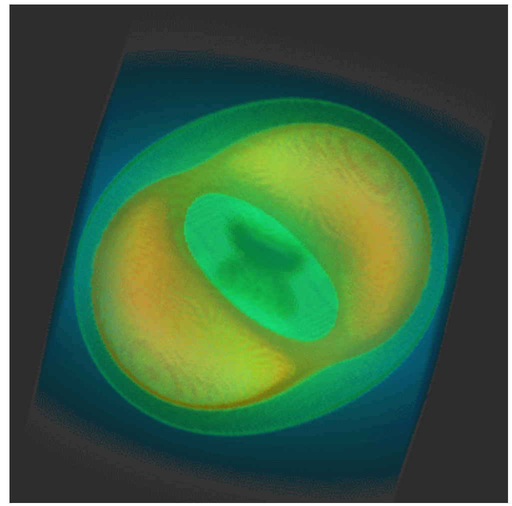
Example: an analytic function

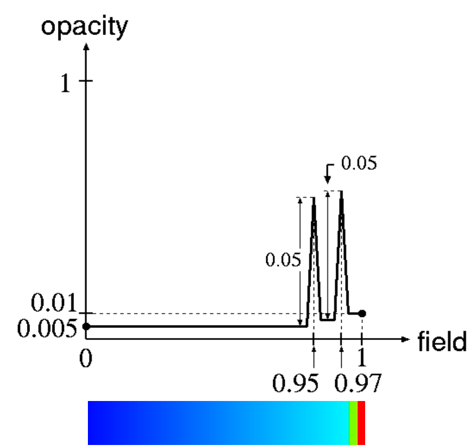
Visualization Result (size 64^3)

Default TF





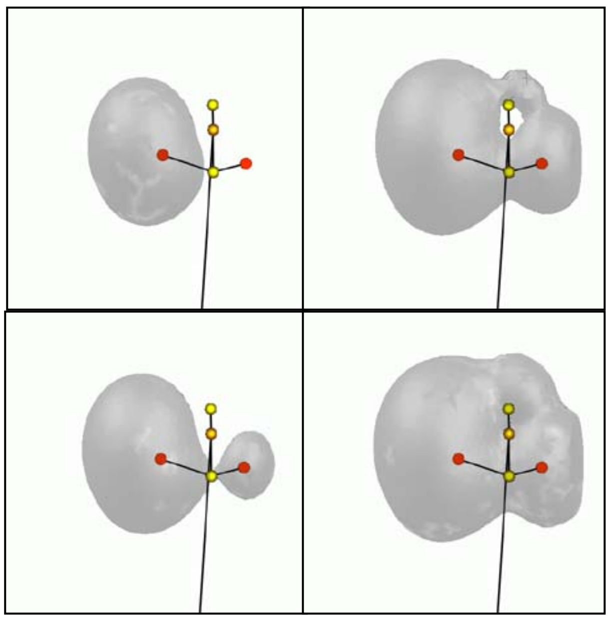




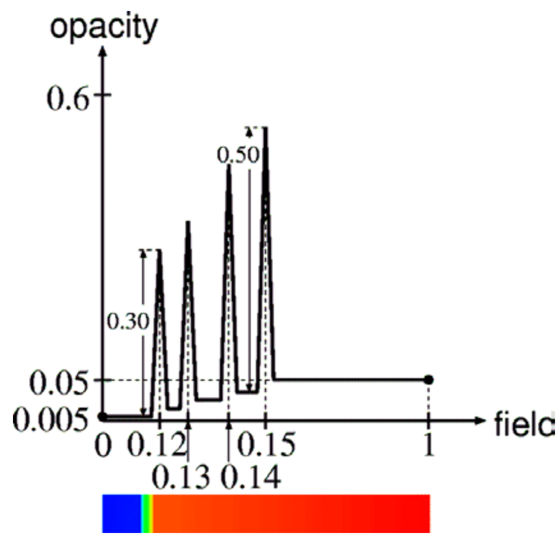
Accentuated TF

Example: Proton and Hydrogen-atom collision

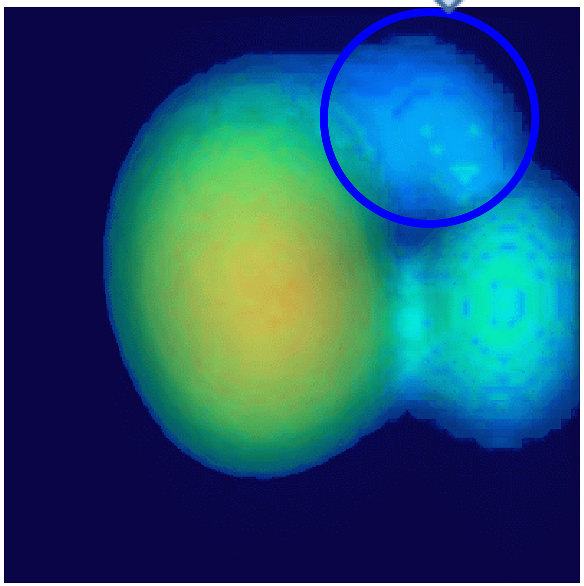
This was found by virtue of the topologically accentuated TF.



Characteristic iso-surfaces



Designed TF



Visualization result

After the Collision

I am now engaged in a joint work with a **steel company** in Japan.

They can get **3D data** of steel materials by taking pictures of a lot of slices. (This is already not so easy!)

↓

We can construct a Reeb graph (contour tree) to visualize the 3D data.

↓

Can be used to estimate certain physical properties of the material (without doing any experiments that cost a lot).

In fact, certain technologies in **Topology** can be also useful!

Homology, Cohomology, Betti Numbers, Euler Characteristics, Persistent Homology, ...

An Example of **MI**

§2. Visualization of Multi-function Data

Fiber

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Definition 2.1 For $c \in \mathbf{R}^m$, $f^{-1}(c)$ is called a **fiber** (rather than a level set).

Example of fibers

$n = 3$, M^3: sea water, $f : M^3 \to \mathbf{R}^2$
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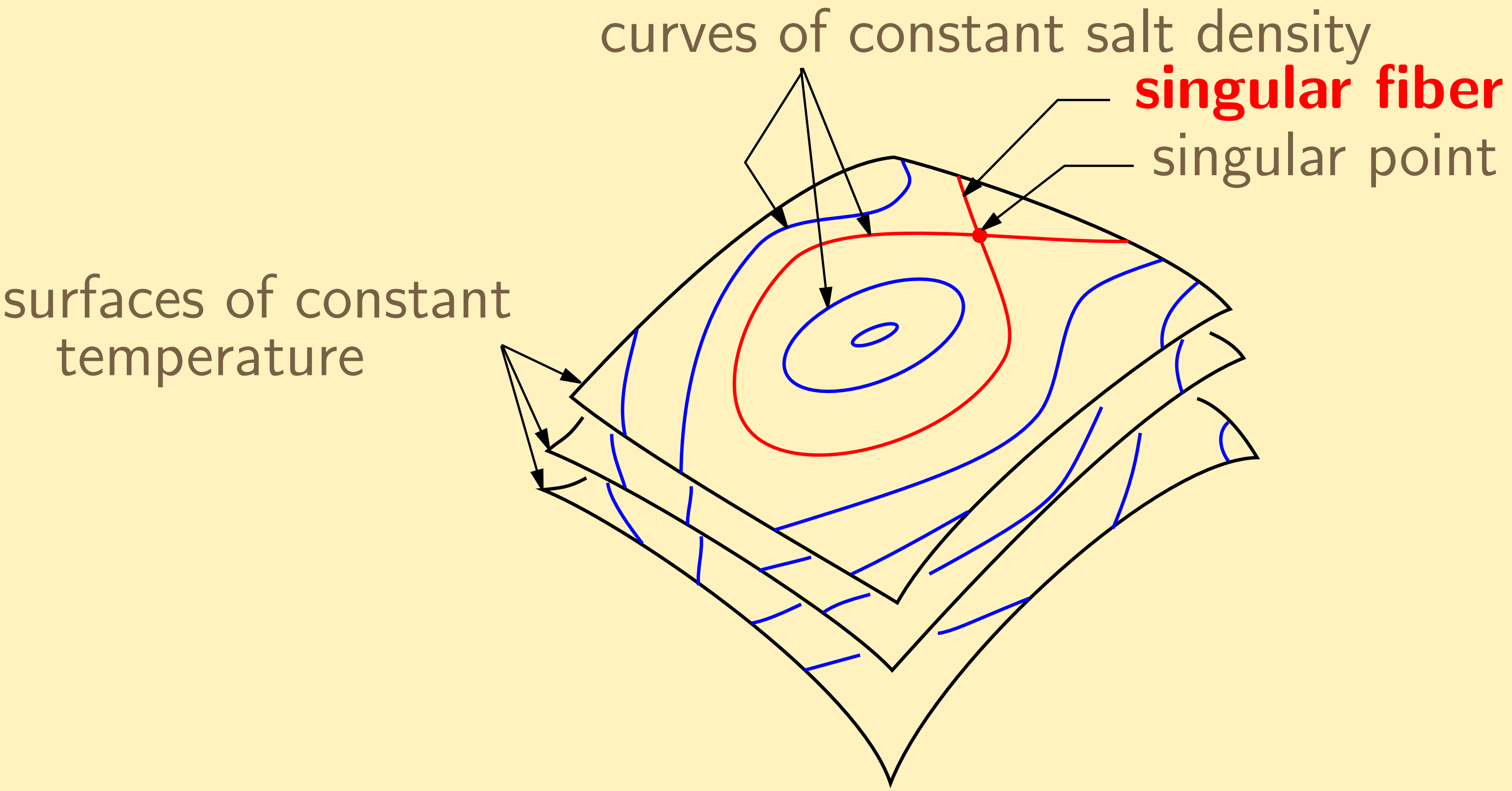

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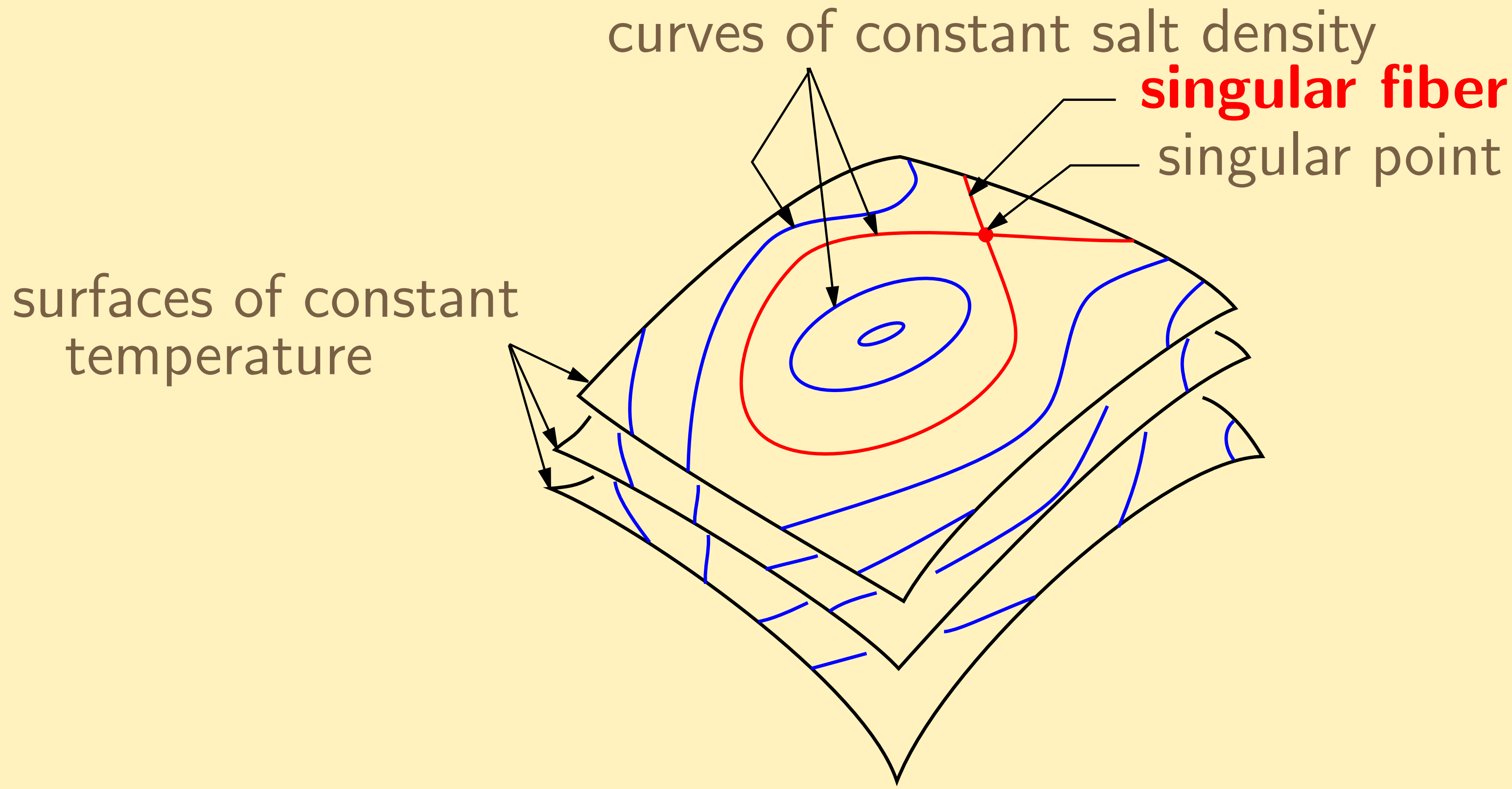
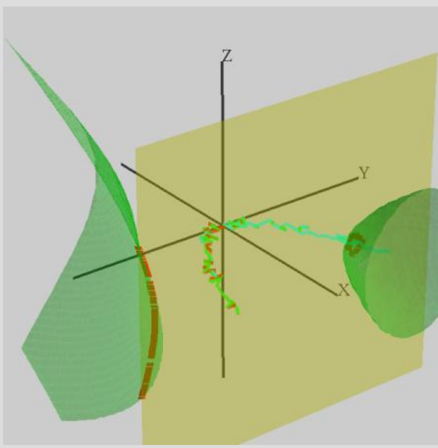


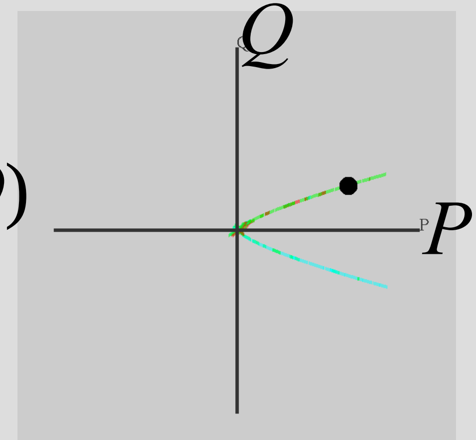
A fiber containing a singular point is called a **singular fiber**.

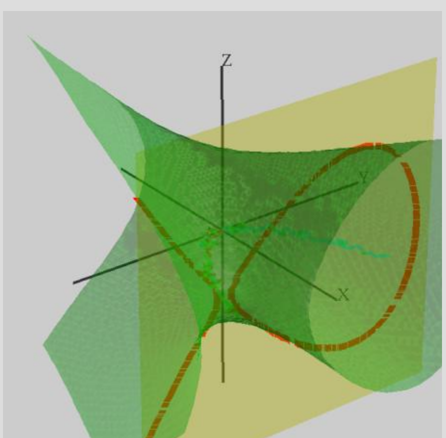
Singular Fibers

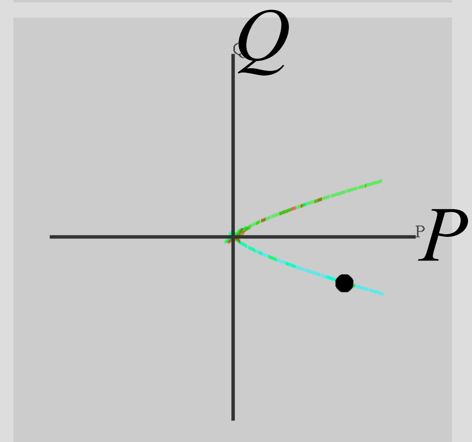
- Topology of the fibers change around singular fibers



$$f : \mathbf{R}^3 \to \mathbf{R}^2$$
$$(x, y, z) \to (P, Q)$$







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Theorem 2.2 (Morse lemma) *If f is* **generic** *enough, then around each critical point, f is expressed as*

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How about the case of multi-functions?

Singular points and Jacobi set

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is the linear map associated with the **Jacobian matrix** of f (the $m \times n$ matrix whose entries are the first order partial derivatives).

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In general, the Jacobi set $J(f)$ is of dimension $m - 1$.

Case of maps into $\mathbb{R}^2$

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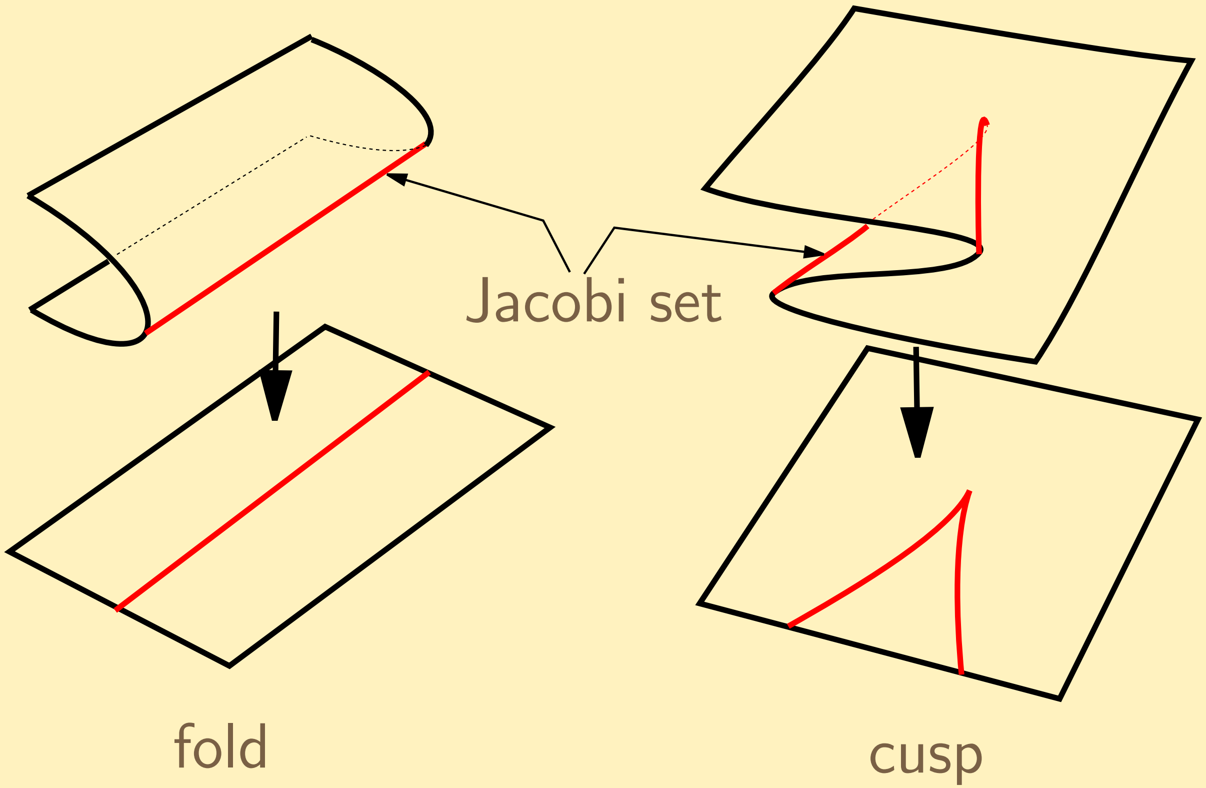

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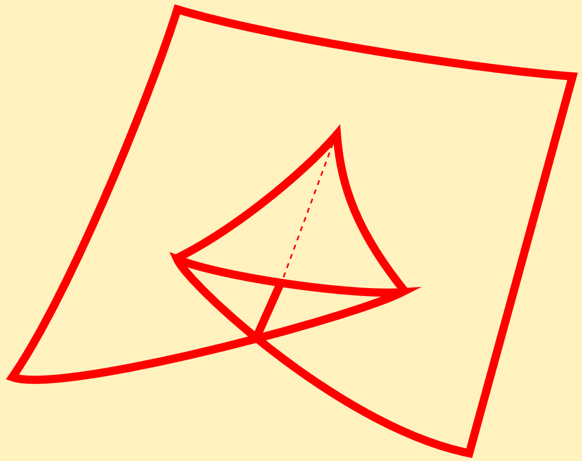

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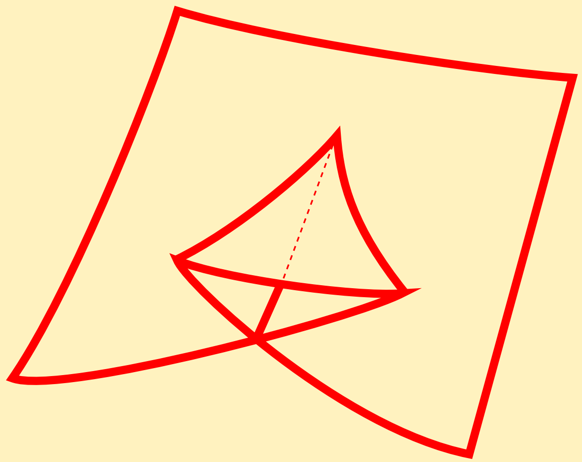

For $m \geq 4$, the situation is much more complicated.

Singular points of multi-functions

$f : M^n \to \mathbf{R}^m$

Suppose $n \geq m = 2$.

fold: A generalization of the Morse critical points for scalar functions

cusp: A degeneration of fold singularities

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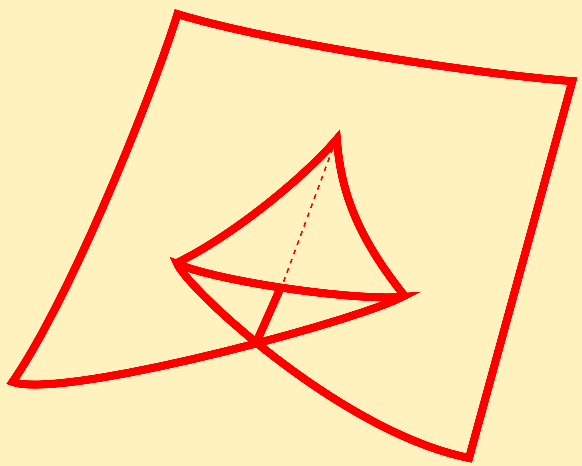

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$\Longrightarrow$ still extensively studied!

Identifying cusps

$f : M^n \to \mathbf{R}^m$

Suppose $n - m > 0$ is odd.

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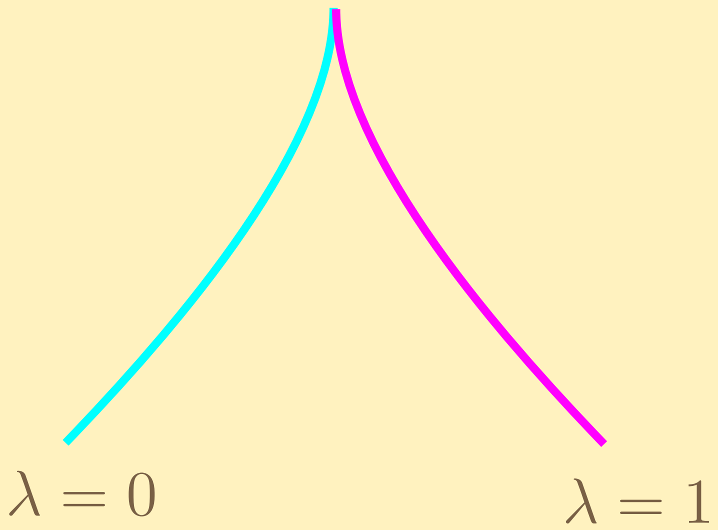

When $n = 3$ and $m = 2$.

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Suggested an algorithm for obtaining the Jacobi set.

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Singularity theory of differentiable mappings

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Singularity theory of differentiable mappings

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If one can identify the folds, cusps and swallowtails, this can contribute a lot to the **visualization** of big data sets.

For visualization

For visualization of multi-function data, we need to

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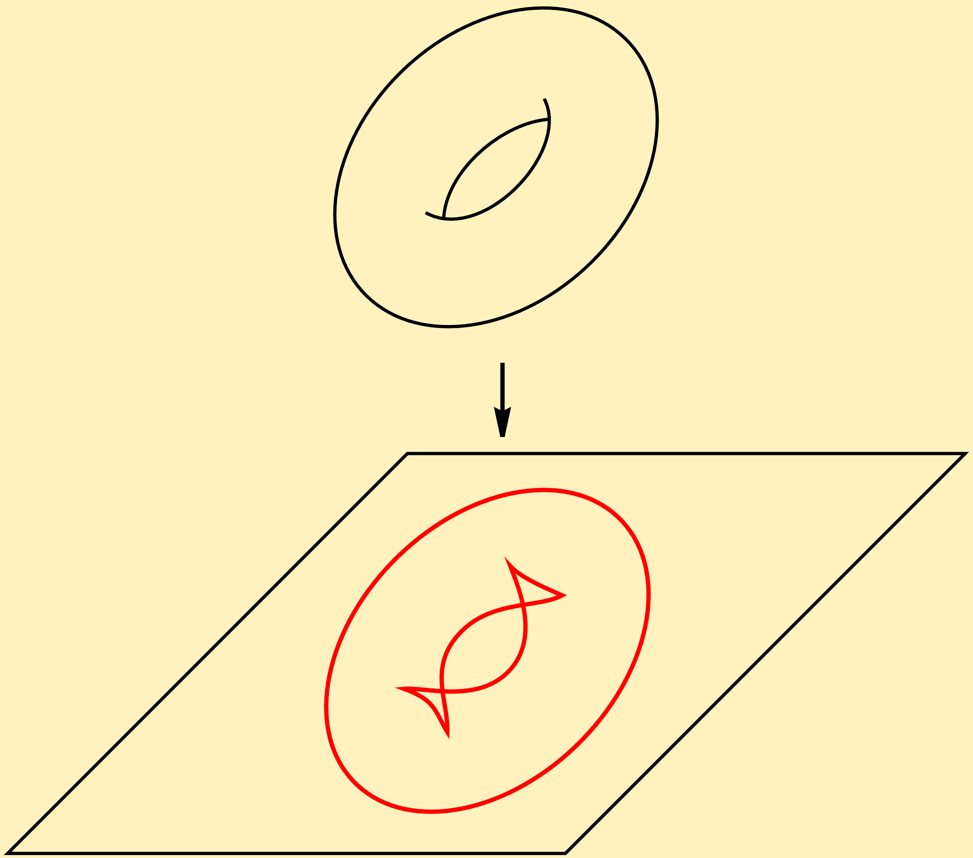
In particular, for item 4 above, it is essential to identify the **singular fibers** and the **fiber changes** near singular fibers.

Example of a Jacobi set image



Jacobi set image of a map of a surface into $\mathbf{R}^2$

When $n = 3$, $m = 2$

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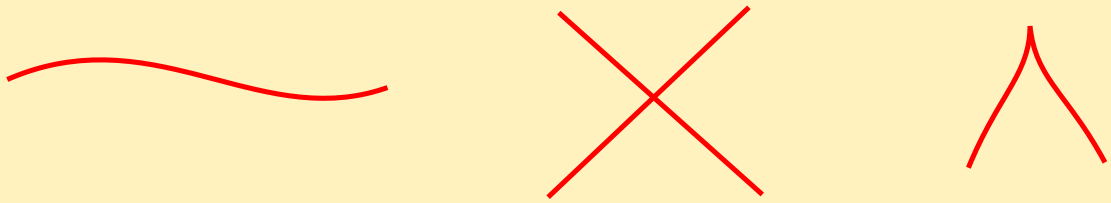

Local configurations of a Jacobi set image

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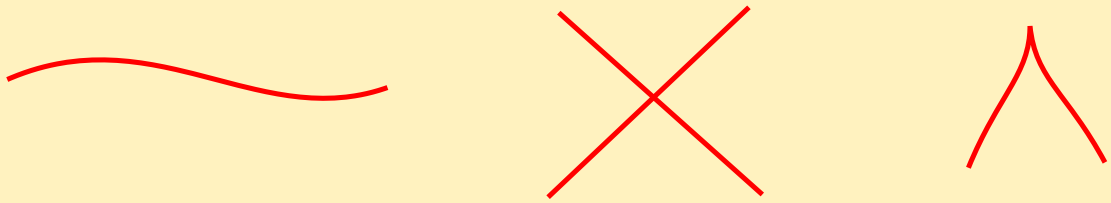

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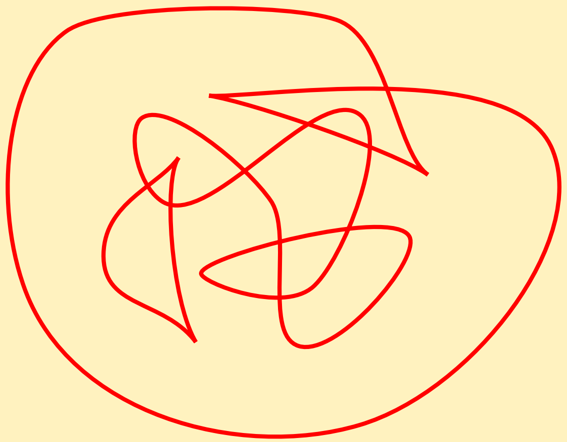
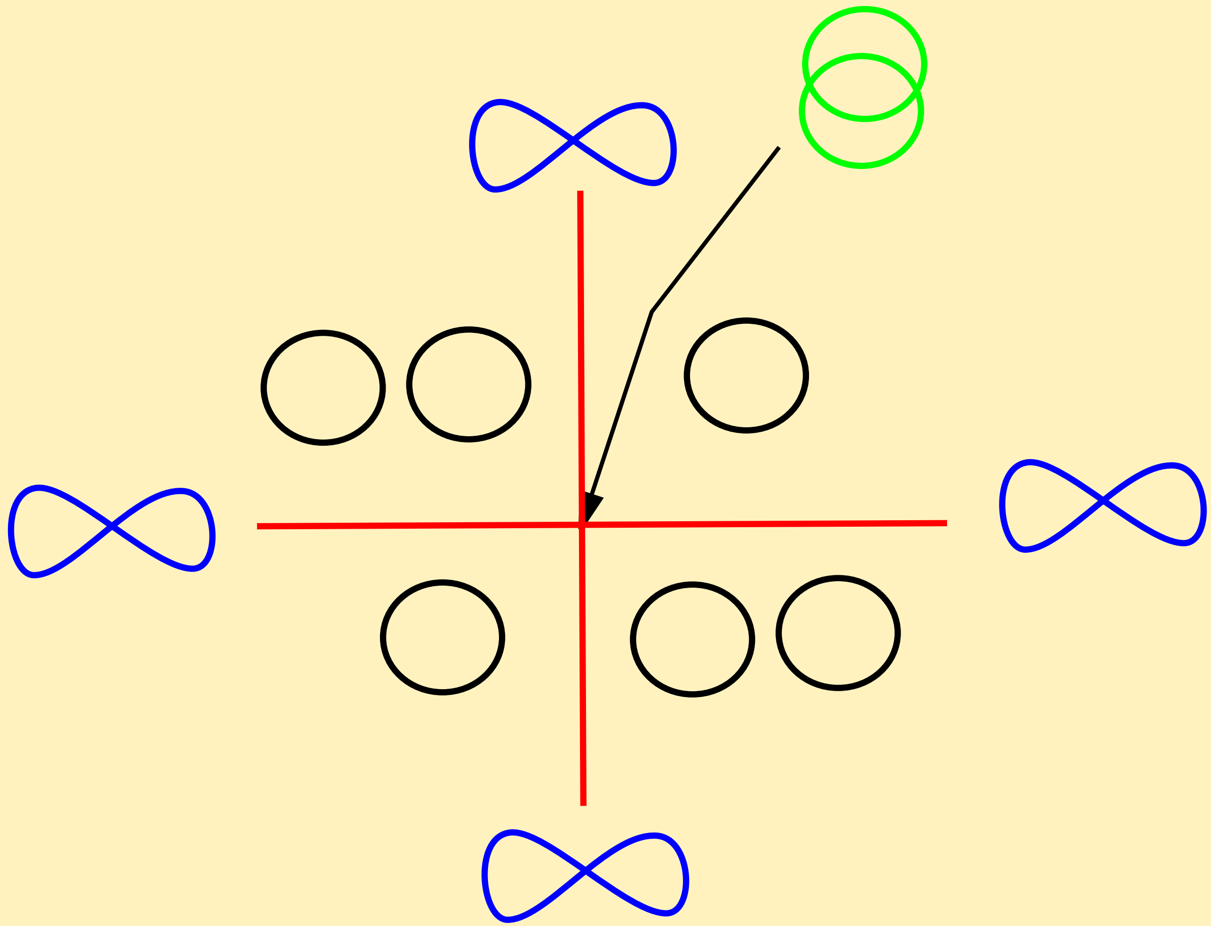


Example of a Jacobi set image

Example of local fiber changes



Fiber over each region of $\mathbf{R}^2 \setminus f(J(f))$

Complexity of a fiber

Fibers are classified into some classes according to their complexities.
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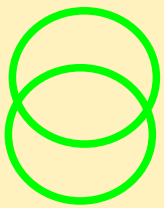
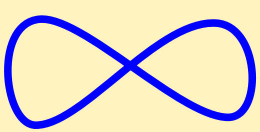
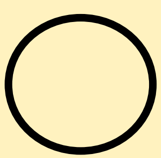


Most complicated, $\kappa = 2$, appears discretely

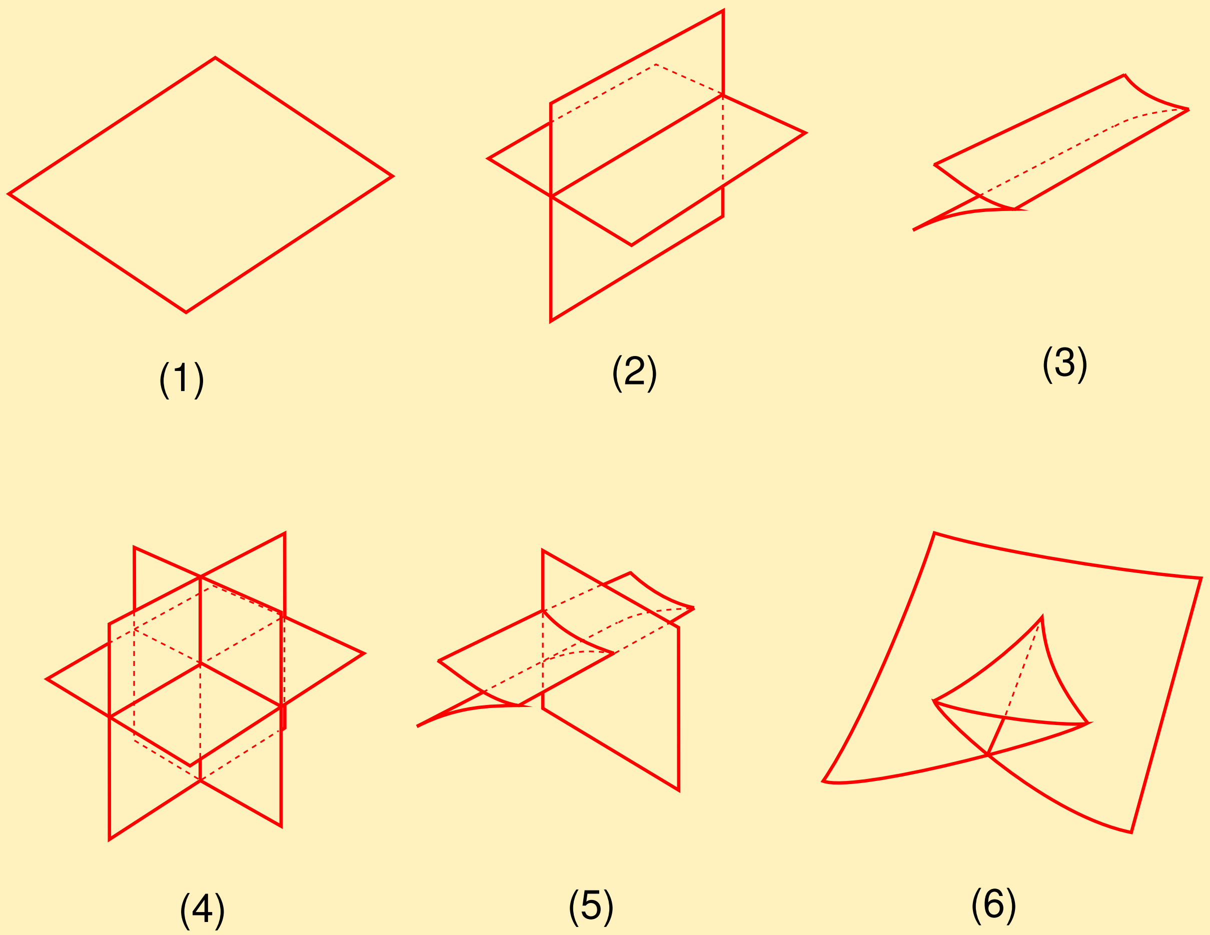


Moderately complicated, $\kappa = 1$, appears along curves



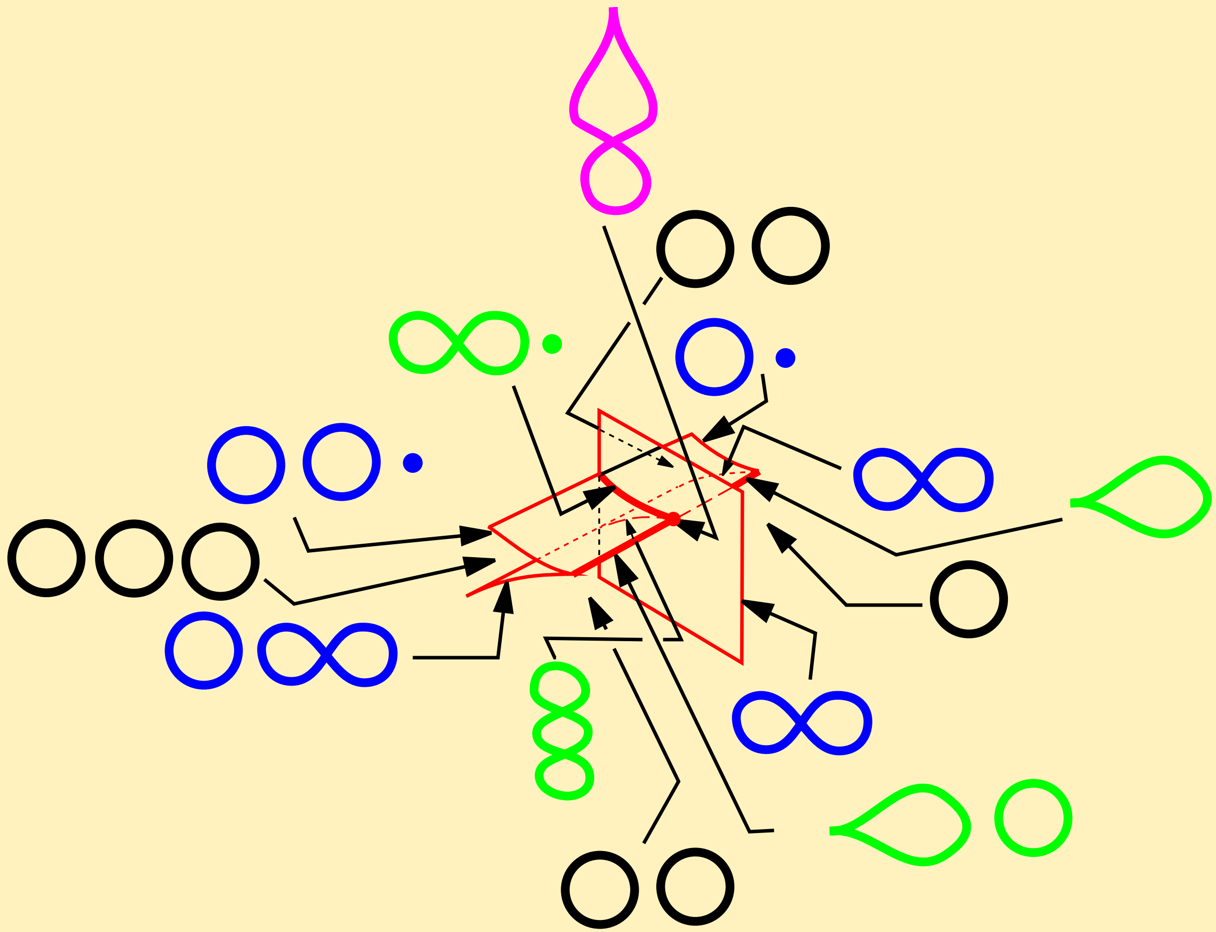
Most simple, $\kappa = 0$, appears along surfaces

When $n = 4,\ m = 3$



Local configurations of the Jacobi set image for maps $f : M^4 \to \mathbf{R}^3$

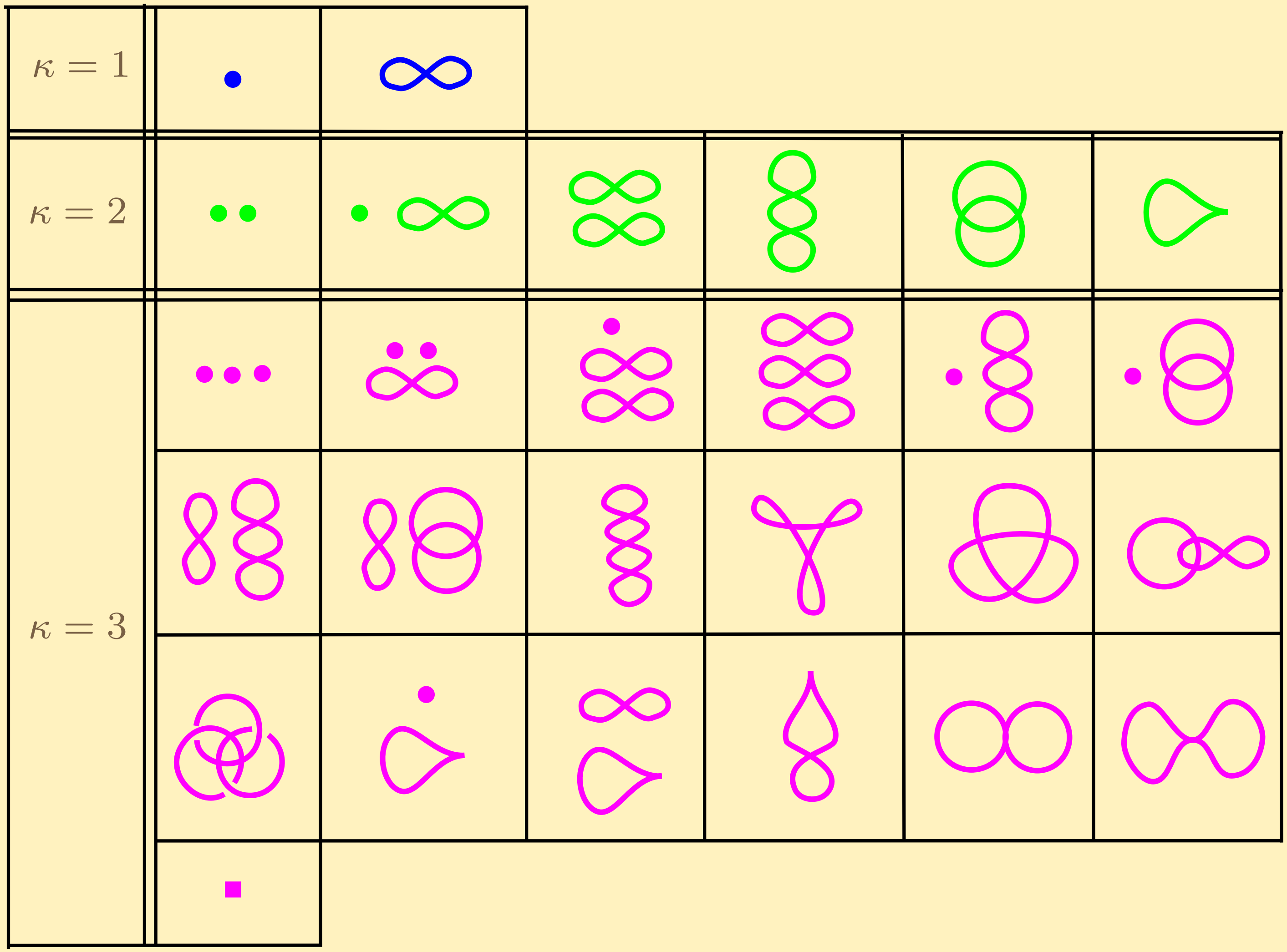
Example of fiber changes



An example of fiber changes for a map $f : M^4 \to \mathbf{R}^3$

List of singular fibers

When $n = 4$, $m = 3$



Conclusion

By using the singularity theory of differentiable mappings,

- We can list up singularity types that appear generically.
- We can list up types of fibers.
- We can identify the singularities (or the singular fibers), and we can determine their types (to a certain extent).

$\Uparrow$

still being investigated

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Muito obrigado!