Free field approach to the Macdonald process

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Macdonald measure (A special case of Macdonald process)

- It is a probability measure on $\mathbb{Y} = \{\text{partitions}\} = \{\text{Young diagrams}\}$.
- The weight for $\lambda \in \mathbb{Y}$ is given by

$$\mathbb{P}_{q,t}(\lambda) \propto P_\lambda(X;q,t)Q_\lambda(Y;q,t),$$

  where $P_\lambda(X;q,t)$ is the Macdonald symmetric function corresponding to λ and $Q_\lambda(Y;q,t)$ is its dual: $\langle P_\lambda(q,t), Q_\mu(q,t)\rangle_{q,t} = \delta_{\lambda\mu}$.
  Ref: A. Borodin and I. Corwin, "Macdonald processes", PTRF **158**, 225-400 (2014).
- Variables $X = (x_1, x_2, \dots,)$ and $Y = (y_1, y_2, \dots)$ are regarded as parameters.
- It reduces to interesting stochastic models by specializing variables.
- A function $f : \mathbb{Y} \to \mathbb{F} = \mathbb{C}(q^{1/2}, t^{1/2})$ is a random variable.
- Problem: compute the expectation value

$$\mathbb{E}_{q,t}[f] := \sum_{\lambda \in \mathbb{Y}} f(\lambda)\mathbb{P}_{q,t}(\lambda).$$

Free field realization

- Let $\mathcal{F}$ be the Fock representation of a deformed Heisenberg algebra:

$$[a_m, a_n] = m\frac{1-q^{|m|}}{1-t^{|m|}}\delta_{m+n,0}, \quad m, n \in \mathbb{Z}\backslash\{0\}.$$

- Space of symmetric functions $\Lambda \simeq \mathcal{F}$ by $p_n \leftrightarrow a_{-n}$, $n \in \mathbb{Z}_{>0}$.
- The Macdonald symmetric functions $\{P_\lambda(q,t) : \lambda \in \mathbb{Y}\}$ are eigenfunctions of a family of commuting operators ($\ni$ Macdonald operators).
- The first Macdonald operator is realized as

$$\hat{E}_1 = \int \frac{dz}{2\pi\sqrt{-1}} \exp\left(\sum_{n>0} \frac{1-t^{-n}}{n} a_{-n} z^n\right) \exp\left(-\sum_{n>0} \frac{1-t^n}{n} a_n z^{-n}\right).$$

- Expectation values are computed only using commutation relations.