

Isomorphism Theorems and the Sign Cluster Geometry of the Gaussian Free Field

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We consider the Gaussian free field (GFF) on a large class of transient weighted graphs G , and prove that its sign clusters contain an infinite connected component. In fact, we show that the sign clusters fall into a regime of strong supercriticality, in which two infinite sign clusters dominate (one for each sign), and finite sign clusters are necessarily tiny, with overwhelming probability. Examples of graphs G belonging to this class include cases in which the random walk on G exhibits anomalous diffusive behavior. Among other things, our proof exploits a certain relation (isomorphism theorem) relating the GFF to Sznitman's random interacements. Our findings also imply the existence of a nontrivial percolating regime for the vacant set of random interacements on G and relate to certain disconnection problems for random walks. Based on joint work with A. Prévost and A. Drewitz (U. Köln).