Determinantal Point Processes and Extrapolation

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Point processes whose correlation functions are given by determinants arise in the study of very different objects such as, for example, random matrices (Dyson), fermions (Macchi), random series (Peres-Virag, Krishnapur), spanning trees (Burton-Pemantle, Benjamini-Lyons-Peres-Schramm), Young diagrams (Baik-Deift-Johansson, Borodin-Okounkov-Olshanski), non-intersecting diffusions (Johansson, Osada-Tanemura), representations of infinite-dimensional groups (Borodin-Olshanski).

For general determinantal point processes a rich general theory has been built, which includes existence theorems (Macchi, Soshnikov, Shirai-Takahashi), description of Palm measures (Shirai-Takahashi, Lyons), rigidity (Ghosh-Peres), the Central Limit Theorem (Soshnikov) and its functional analogue (joint work with Dymov).

The course will start with an elementary introduction to determinantal point processes, proceeding to recent developments and open problems.

For example, consider a Gaussian Analytic Function on the disk, that is, a random series whose coefficients are independent complex Gaussians. In joint work arXiv:1612.06751 with Yanqi Qiu and Alexander Shamov, we show that the zero set of a Gaussian Analytic Function is a uniqueness set for the Bergman space on the disk: in other words, almost surely, there does not exist a nonzero square-integrable holomorphic function having these zeros. The key rôle in our argument is played by the determinantal structure of the zeros due to Peres and Viràg. In general, we prove that the family of reproducing kernels along a realization of a determinantal point process generates the whole ambient Hilbert space, thus settling a conjecture of Lyons and Peres.

The key lemma in our argument is that the determinantal property is preserved under conditioning on a fixed restriction of our configuration in a part of the phase space. In full generality, the explicit description of the kernels of these conditional measures remains an open problem.

It is nevertheless possible to write the analogue of the Gibbs property for one-dimensional determinantal point processes with integrable kernels: for these processes, conditional measures with respect to fixing the configuration in the complement of an interval is an orthogonal polynomial ensemble with explicitly found weight (arXiv:1605.01400). Note here that a reproducing kernel must have integrable form as soon as the corresponding Hilbert space satisfies a weak form of division property (joint work with Roman Romanov); in this case, our space must be a Hilbert space of holomorphic functions.

The proof of the Lyons-Peres conjecture raises the problem of extrapolating a function from its restriction to a realization of a determinantal point process. In joint work with Yanqi Qiu (arXiv:1806.02306), extrapolation for Bergman functions from zero sets of Gaussian Analytic Functions is obtained using the Patterson-Sullivan construction.