Isomorphisms between determinantal point processes and Poisson point processes

Shota OSADA *Kyushu University, Japan*, E-mail: s-osada@math.yushu-u.ac.jp

Abstract:

We study an isomorphism problem between translation invariant determinantal point processes (DPPs) and Poisson point processes (PPPs) in the sense of ergodic theory.

In measure-preserving dynamical systems (m.p.d.s.), two m.p.d.s. $(X_1, \mathcal{F}_1, \mathbb{P}_1, \mathsf{T}_1)$ and $(X_2, \mathcal{F}_2, \mathbb{P}_2, \mathsf{T}_2)$ are said to be isomorphic if there is a bi-measurable bijection $\phi : X_1 \to X_2$ such that $\mathbb{P}_1 \circ \phi^{-1} = \mathbb{P}_2$ and $\phi \circ \mathsf{T}_1 = \mathsf{T}_2 \circ \phi$. Let ν be a translation invariant process indexed by $\mathbb{Z}^d$. We call ν Bernoulli if ν is isomorphic to an *i.i.d.* process. In the isomorphism problem, Bernoulli is a special class where Kolmogorov-Sinai entropy is a complete invariant of isomorphisms. In translation invariant processes indexed by $\mathbb{R}^d$, Poisson point processes play a role of Bernoulli shifts with infinite entropy because Poisson point processes have no interaction between particles.

DPPs and PPPs greatly differ in terms of interaction among particles. For instance, a rigidity of determinantal point processes has proved in [1,3,4,6]. We prove, however, DPPs and PPPs are isomorphic.

Theorem. *Let μ^K be a determinantal point process on $\mathbb{R}^d$ with a translation invariant kernel $K(x,y) = k(x-y)$. Assume*

$$k(x) = \int_{\mathbb{R}^d} \hat{k}(t) e^{2\pi i x \cdot t} dt$$

for some $\hat{k} \in L^1(\mathbb{R}^d, [0,1])$. Then μ^K is isomorphic to a Poisson point process.

Translation invariant DPPs on $\mathbb{Z}^d$ have been proved to be Bernoulli [8,9]. However, isomorphisms between DPPs on $\mathbb{R}^d$ and Poisson point processes have not been studied. Our theorem also provides another proof of tail triviality of μ^K, which is a partial result of [2,5,7].

References

[1] Bufetov, A. I. (2016). Rigidity of determinantal point processes with the Airy, the Bessel and the Gamma kernel *Bull. Math. Sci.*, **6**, 163–172.

[2] Bufetov,A. I., Yanqi Qiu and Shamov, A. (2016) Kernels of conditional determinantal measures and the Lyons-Peres Conjecture (arXiv:1612.06751).

[3] Ghosh, S (2016). Palm measures and rigidity phenomena in point processes *Electron. Commun. Probab.*, **21**, 1–14.

[4] Ghosh, S. and Peres, Y. (2017) Rigidity and tolerance in point processes: Gaussian zeros and Ginibre eigenvalues *Duke Math. J.*, **166**, 1789–1858.

[5] Osada, H. and Osada, S. (2018). Discrete approximations of determinantal point processes on continuous spaces: tree representations and the tail triviality *J Stat Phys*, **170**, 421–435.

[6] Osada, H. and Shirai, T. (2016). Absolute continuity and singularity of Palm measures of the Ginibre point process *Probab. Theory Relat. Fields*, **165**, 725–770.

[7] Lyons, R. (2018) A Note on Tail Triviality for Determinantal Point Processes (arXiv:1807.06167).

[8] Lyons, R. and Steif, J. E. (2003). Stationary determinantal processes: phase multiplicity, bernoullicity, and domination *Duke. Math. J.*, bf 120 , 515–575.

[9] Shirai T. and Takahashi Y. (2003). Random point fields associated with certain Fredholm determinants II: fermion shifts and their ergodic properties, *Ann. Prob.*, **31**, 1533–1564.