On uniqueness of Dirichlet forms related to interacting systems with an infinite number of particles

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An infinite system of interacting Brownian motions in $\mathbb{R}^d$ can be represented by a stochastic process on the (unlabeled) configuration space $S$, the set of integer valued radon measures on $\mathbb{R}^d$. This process can be constructed by means of several probabilistic argument. Among them, Osada [2] constructed the processes in general setting by using Dirichlet form technique. For a suitable probability measure $\mu$ on $S$, the Dirichlet form $(\mathcal{E}^\mu, \mathcal{D}^\mu)$ introduced in [2] is obtained by the smallest extension of the bilinear form $(\mathcal{E}^\mu, \mathcal{D}^\mu)$ on $L^2(S, \mu)$ with domain $\mathcal{D}^\mu_0$ defined by

$$\mathcal{E}^\mu(f, g) = \int_S \mathcal{D}[f, g](s) \mu(ds), \quad \mathcal{D}[f, g](s) = \frac{1}{2} \sum_{i=1}^{\infty} \nabla s_i \cdot \nabla s_i \mathcal{g},$$

$$\mathcal{D}^\mu_0 = \{ f \in \mathcal{D}_0 \cap L^2(S, \mu); \mathcal{E}^\mu(f, f) < \infty \},$$

where $\mathcal{D}_0$ is the set of all local smooth functions on $S$, $\mathcal{f}$ is a symmetric function such that $\mathcal{f}(s_1, s_2, \ldots) = f(s)$, $\cdot$ is the inner product in $\mathbb{R}^d$, and $s = \sum_i \delta_{s_i}$ denotes a configuration. This Dirichlet form is the decreasing limit of the Dirichlet forms associated with finite systems of interacting Brownian motions in bounded domains $S_R = \{ x \in \mathbb{R}^d; |x| \leq R \}$ with a boundary condition. Because of the boundary condition, when a particle touches the boundary, it disappears. And particles enter the domain from the boundary according to the reversible measure $\mu$.

On the other hand, Lang [1] constructed the infinite system of Brownian motions as a limit of stochastic dynamics in bounded domains $S_R$ by taking the finite systems with another boundary condition. In his finite systems when a particle hits the boundary, it reflects and the number of particles in the domain are invariant. His process associated with the Dirichlet forms $(\mathcal{E}^\text{lor}, \mathcal{D}^\text{lor})$ the increasing limit of the Dirichlet forms associated with them.

In this talk, we discuss the relation between these Dirichlet forms $(\mathcal{E}^\text{upr}, \mathcal{D}^\text{upr})$ and $(\mathcal{E}^\text{lor}, \mathcal{D}^\text{lor})$. The main purpose of this paper is to give a sufficient condition for

$$(\mathcal{E}^\text{lor}, \mathcal{D}^\text{lor}) = (\mathcal{E}^\text{upr}, \mathcal{D}^\text{upr}).$$

References
