Stochastic complex Ginzburg-Landau equation with space-time white noise

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This is a jointwork with Masato Hoshino (Waseda University) and Nobuaki Naganuma. The preprint is uploaded on arXiv Preprint Server (arXiv:1702.07062).

The main objective of this talk is to prove local well-posedness of the stochastic cubic complex Ginzburg-Landau equation on the three-dimensional torus $T^3 = (\mathbb{R}/\mathbb{Z})^3$ of the following form by using Gubinelli-Imkeller-Perkowski’s paracontrolled calculus:

$$\partial_t u = (i + \mu)\Delta u + \nu(1 - |u|^2)u + \xi, \quad t > 0, \quad x \in T^3.$$

Here, $i = \sqrt{-1}$, $\mu > 0$, $\nu \in \mathbb{C}$ are constants and $\xi$ is complex-valued space-time white noise, that is, a centered complex Gaussian random field with covariance

$$\mathbb{E}[\xi(t, x)\xi(s, y)] = 0, \quad \mathbb{E}[\xi(t, x)\overline{\xi(s, y)}] = \delta(t - s)\delta(x - y),$$

where $\delta$ denotes the Dirac delta function.

We approximate $\xi$ by a smeared noise $\xi^\epsilon$ with a parameter $0 < \epsilon < 1$ (by killing the high frequencies) so that $\xi^\epsilon \rightarrow \xi$ as $\epsilon \downarrow 0$ in an appropriate topology. We consider a renormalized equation

$$\partial_t u^\epsilon = (i + \mu)\Delta u^\epsilon + \nu(1 - |u^\epsilon|^2)u^\epsilon + \nu C^\epsilon u^\epsilon + \xi^\epsilon, \quad t > 0, \quad x \in T^3,$$

where $C^\epsilon$ is a suitably chosen complex constant which diverges as $\epsilon \downarrow 0$. We show that the solution $u^\epsilon$ converges to a certain non-trivial process in an appropriate topology.

Unlike regularity structure theory, there is no “bible” for paracontrolled calculus, since it has been developed gradually. For the probabilistic part, we follow Gubinelli-Perkowski’s method (2017, CMP). For the deterministic part, we follow Mourrat-Weber’s method (2017+, AoP).

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