A central limit theorem for stochastic heat equation in random environment

Lu Xu (The University of Tokyo)
xltodai@ms.u-tokyo.ac.jp

Consider a stochastic heat equation on $[0,1]$ with homogenous Neumann boundary conditions and a random non-linear term

$$\partial_t u^\sigma(t,x) = \frac{1}{2}\partial_x^2 u^\sigma(t,x) - U(\sigma, u^\sigma(t)) + \dot{W}(t,x),$$

$$\partial_x u^\sigma(t,x)|_{x=0} = \partial_x u^\sigma(t,x)|_{x=1} = 0,$$

where $\dot{W}(t,x)$ is a standard space-time white noise on some probability space $(\Omega, \mathscr{F}, \mathbb{P})$ and $U : \Sigma \times L^2[0,1] \to L^2[0,1]$ is a random field defined on some other space $(\Sigma, \mathscr{A}, \mathbb{Q})$.

We suppose that $\mathbb{Q}$-almost every sample path of U is bounded and Lipschitz continuous, and can be decomposed into the following form:

$$U(\sigma, u) = DV(\sigma, u) + B(\sigma, u),$$

where $V(\sigma, u) \in \mathbb{R}$ and D stands for the Fréchet differentiable operator on u. Assume that V and B satisfy the following divergence-free condition:

$$E_{\mu_0}\left[e^{-2V(\sigma,\cdot)}\langle Df, B(\sigma,\cdot)\rangle_{L^2[0,1]}\right] = 0, \quad \forall f \in C_b^1(H;\mathbb{R}),$$

where μ_0 is the standard Wiener measure on $L^2[0,1]$. Also assume that the environment U is stationary and ergodic under the transfer group $\{\tau_c; c \in \mathbb{R}\}$ defined by

$$\tau_c \phi \triangleq \phi(\cdot + c\mathbf{1}), \quad \forall \phi : H \to H,$$

where $\mathbf{1}$ stands for the constant function $\mathbf{1}(x) \equiv 1$.

We extend the method of recording the environment viewed from the observer to our non-linear setting, and prove a central limit theorem for $u^\sigma(t,\cdot)/\sqrt{t}$, which holds in probability with respect to $\mathbb{Q}$. The limit is a centered Gaussian law concentrating only on constant functions.