On loops of Brownian motion

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Abstract

In this talk, we will consider the nature of self-intersections of the Brownian motion in $\mathbb{R}^d$. It is well-known that the Brownian motion is a simple path if $d \geq 4$, while it has loops when $d \leq 3$. What are loops of the Brownian motion? How are those loops distributed in space? These are questions we want to address. In the talk, we give an explicit representation of such loops for $d \leq 3$ by establishing a decomposition of the Brownian path into independent simple path and a set of loops. It turns out that the simple path and the set of loops can be described by the scaling limit of the loop-erased random walk and a Poisson point process on a path space, respectively. We will also explain a relation between our results and Itô's excursion theory.

This is joint work with Artem Sapozhnikov (University of Leipzig).